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Higgs sector from a non-universal $U(1)_X$ supersymmetric model

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Abstract

The project consider a $U(1)_X$ non-universal extension to the Minimal Supersymetric Standard Model for explaining more naturally the fermion mass hierarchy, in the sense that Yukawa couplings with the scalar background are at the same order of magnitude. The hierarchy feature relies on a more complex scalar potential than in the Minimal Supersymmetric Standard Model, involving more scalar fields with different vacuum expectation values. The model also predicts the 125 GeV Higgs boson with no need of large radiative corrections, if the set of parameters of the model is constrained. The scalar spectrum is increased compared to the Minimal Supersymmetric Standard Model. These hypothetical particles are explained in the model to be arbitrarily heavy, escaping the experimental detection so far.

Publications

Many of the results presented in the chapter 5 were shown previously in the following publication:

- Nonuniversal $U(1)_X$ extension to the MSSM with three families [1].

Results from this work were also shown in some conferences:

- A non-universal $U(1)_X$ gauge extension to the MSSM [2]. Prented at: Meeting of the Division of Particles and Fields of the American Physical Society (DPF2019).
- Constaints for a Z' boson with non-universal couplings in a supersymmetric model. Presented at: 4th Colombian Meeting on High Energy Physics, 2019.

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List of abbreviations

SUSY: Supersymmetry
SM: Standard Model
MSSM: Minimal Supersymmetric Standard Model
NMSSM: Next to Minimal Supersymmetric Standard Model
VEV: Vacuum expectation value
FMH: Fermion mass hierarchy
SSM: Seesaw mechanism
ISSM: Inverse seesaw mechanism
NWA: Narrow Width Approximation

Chapter 1

Introduction

The most successful model that explains the fundamental particles is the Standard Model (SM) from A. Salam, L. Glashow and S. Weinberg [3–5]. It is a quantum field theory that contains fermion fields, gauge boson fields and a $SU(2)$-doublet scalar field, all interacting together in a renormalizable framework. The model is invariant under the Poincaré symmetry, which is a requirement for the model to be consistent with special relativity, and an internal symmetry $SU(3) \times SU(2)_L \times U(1)_Y$, that provides a description of the fundamental forces. The agreement with experimental results has been very strong until today [6]. The last piece of the puzzle to be experimentally observed was the Higgs boson, the massive particle related to the scalar field. Its mass has been reported to be around 125 GeV [7, 8]. However, there are certain features that may indicate a path towards physics beyond the energy scales that have been reached so far. Besides the inclusion of gravity (or even the inclusion into gravity), perhaps the most mysterious one is the explanation of the rest mass of the particles observed so far. The way that it is explained in the SM is through the the potential of the scalar field, since it does not have an unique minimum and therefore the symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken into the known quantum electrodynamics $U(1)_Q$. Thereby, goldstone modes are interpreted as the mass of the massive interaction bosons W and Z of the electroweak force. This is known as the *Higgs Mechanism*. The Higgs mass is directly explained by this mechanism and its own potential. The mass of the fermions comes from the interaction of the fermion fields with the scalar field, which in the process of spontaneous symmetry is regarded as the rest mass. Many questions from here have arisen in the field, and they have lead possible clues for physics beyond the standard model.

One of them comes from the fact that, if the Standard Model is regarded as an effective field theory, the square of the Higgs' mass depends quadratically on the cutoff energy. This means that the mass of the Higgs is sensitive to the physics that is not yet known. Therefore, the effective field theory loses its predictability [9]. This is a *naturalness* problem that is called *the hierarchy problem*. This has motivated supersymmetry [10], where a symmetry between fermions and bosons cancels the quadratic dependence of the Higgs mass with physics not yet observed. For this purpose comes the Minimal Supersymmetric Standard Model (MSSM) [11], which is the simplest consistent supersymmetric extension that can be made to the SM. Explicitly, it contains all the fields of the SM and an additional scalar doublet, plus a whole sector of fields related by supersymmetry. The solution of the problem really brought many more into the field. The most direct one is that the particle states, that are related to the already measured ones by supersymmetry, have not been yet observed. Thus, the symmetry is not exact and non-supersymmetric behaviors have to be regarded in order to explain the non-observation of those particles. This is

known as soft supersymmetry breaking. Possible explanations of the origin of the breaking of supersymmetry have been widely considered, for instance from supergravity [12], but for the moment they are still a description with a huge quantity of free parameters. Another problematic implication is the violation of the baryon and lepton numbers, which are so far forbidden [13,14]. This is solved, but put by hand, by the inclusion of a parity symmetry called R-parity [15–18], where baryon and lepton numbers are conserved. Another disadvantage of the MSSM is is a naturalness problem that exists in the scalar potential of the MSSM, where for guaranteeing that the scalar fields have a non-zero vacuum expectation value, a μ parameter with mass dimension equal to one should lie at the energy scale of supersymmetry breaking. But since it comes from a term that preserves supersymmetry, the relation is not understood. This is known as *μ-problem* [19,20]. Finally, the protection of the Higgs mass under high energy physics leaves the MSSM with some tensions with the reported 125 GeV scalar resonance. That means that since the upper bound on the Higgs mass in a MSSM is around 130 GeV [21,22], then the measured one is close to it.

Extensions to the MSSM have been proposed to solve or at least to lessen the issues of the model itself. One of them is the Next to Minimal Supersymetric Standard Model (NMSSM), and it includes also a scalar field with its supersymmetric partner [23]. A motivation for this non-minimal extension has been to generate a mass term like the μ-term with the vacuum expectation value of the new scalar field. Those models make the set of parameter even bigger, so the fine-tuning is softened in explaining the Higgs mass [24–29]. Another kind of extension to the MSSM adds a new gauge symmetry $U(1)_X$ [30–33]. Such models are mainly built up in order to provide an effective R-parity conservation. This is done through a non-universal assignation of the charges for the fields under a $U(1)_X$ group. The μ-problem is also solved with a scalar field as in the NMSSM, but here this scalar field is also responsible of the spontaneous symmetry breaking of $U(1)_X$. The gauge extensions of the MSSM can also surpass the MSSM bounds on the Higgs mass.

Leaving supersymmetry aside for the moment and going back to the mass generation of the SM, the explanation of the fermions masses has been also put at debate. Observations of neutrino oscillation [34,35] have demonstrated that neutrinos have mass, although in the SM they happen to be massless. An explanation for their mass is usually done with the inclusion of massive Majorana right-handed neutrinos, too heavy to escape experimental and to account for the smallness of the neutrino masses [36]. This is known as the seesaw mechanism (SSM) [37] and the scale of energy of right-handed neutrinos happens to be between 10^{12} and 10^{15} GeV [38], which is untestable with the current tecnology. An extension that can be regarded to the seesaw mechanism includes as well Dirac right-handed neutrinos, involving an intermediate energy scale closer to current experimental capabilities [39, 40]. This is known as the inverse seesaw mechanism (ISSM) [41]. A naturalness problem is also present, and it is the arbitrariness of the Yukawa couplings for explaining the broad spectrum of fermion masses, which starts in neutrinos at scales below hundreds of meV [42] and ends in the mass of the top at around 173 GeV [43]. This is known as the fermion mass hierarchy (FMH) problem [44]. In other words, is regarded as a mystery the patterns of the fermionic mass spectrum. Yukawa couplings should also explain the experimental data on flavor changing neutral currents, parameterized in the CKM and PMNS matrices [45–48]. The *flavor problem* regards all these aspects, that in an specific gauge model have to do with the Yukawa couplings [49]. Grand

unification theories (GUT) tried to embrace the FMH through spontaneous symmetry breaking [50–52], however, they did not predict the big mass of the top quark within the *little hierarchies*, as they called the mass hierarchy of the SM particles. A more successful approach is to consider discrete symmetries that constraint Yukawa terms and imply a more natural framework even with fewer parameters. The advantage of this is the freedom of imposing the values of the fields under the discrete symmetries, without the jeopardizing quantum anomalies [53–55]. However, they lack of a clear phenomenology. Another way to attain a FMH has been by the inclusion of a new gauge symmetry $U(1)_X$, whose particles' content and charge distribution makes the model anomaly free [56–58]. In such models, the approach for solving the FMH is to consider more than one scalar field, relying the spectrum in a hierarchized set of vacuum expectation values. A second benefit from this is that the symmetry itself forbids some Yukawa couplings, making the FMH in the model more natural. Imposing a free anomaly model usually requires an extended fermion content of the model with an exotic fermion sector, whose fields are scalar in $SU(2)_L$ because they should not couple to the Higgs boson. The particles of this new sector should be heavy enough to escape experimental constraints. Therefore, their mass is instead explained by Yukawa couplings with a scalar singlet field whose vacuum expectation value does not have upper bounds. The scalar singlet field, which is not considered in the SM, is the responsible for breaking the new symmetry $U(1)_X$. Similarly to the Highs mechanisms, gauge bosons give mass to the not yet observed Z' bosons. An advantage of a model with exotic fermions is the explanation of the rare B meson decays $B \to \bar{K}^0\mu^+\mu^-$ and the Higgs decay $h \to \mu\tau$, forbidden in the standard model [59]. However, such discrepancies can be mere statistical errors [60].

In physics BSM it is important to have observables that can be at least constrained by experimental data, otherwise the exercise falls entirely in the speculative framework. $U(1)_X$ gauge extended models imply the existence of a massive interaction boson Z', whose hypothetical physics can be constrained considering experimental data. For instance, data on dark matter can indirectly give a possible region of the parameter space in Z' physics [33]. Constraints may come also from the Large Hadron Collider (LHC), more recently during its second run. Analyses on diboson and dilepton production on CMS and ATLAS detectors have given lower bounds on the mass of the Z' boson [61–63] for different types of models.

The aim of the present thesis is to describe the Higgs sector from a supersymmetric non-universal $U(1)_X$ extension to the Standard Model. Specifically, the main objective is to find the spectrum of the scalar particles and constraint the parameter space through the explanation of the observed 125 GeV Higgs boson and the data on diboson and dilepton production in the LHC. The model itself is based in another non-supersymmetric one, introduced by S. F. Mantilla and R. Martínez [64]. It is a non-universal $U(1)_X$ model, which regards as well two scalar doublets and two scalar singlets. An exotic fermion sector is also needed for cancelling chiral anomalies. The model explored here is a minimal supersymmetric extension of the previous one. Although the quantity of unresolved issues will still be huge, some possible paths to follow in the exploration of this model will be mentioned. In chapter (2) I give a small review of the field theory necessary for understanding the basics of the Standard Model. Chapter 3 reviews the Standard Model, focusing in the mass generation mechanism. In the same chapter, the discussion is headed towards $U(1)_X$ extensions, which could be used to understand the broad fermion mass spectrum. Next, the work bends itself into another direction in chapter 5 to explain basic features of

supersymmetry. It starts by looking at some motivations towards supersymmetry, where the naturalness concept becomes important. Then it reviews the framework for a superfield theory in order to introduce the supersymmetric extension to the standard model, allowing also an understanding of the motivations for alternatives to this sypersymmetric version. In this way it is possible to present the model in chapter 6, taking a look to the scalar sector which is involved in the Higgs particle mass. LHC and ATLAS data on dilepton and diboson production is compared with the model in order to give restrictions in the parameter space.

Chapter 2

Field theory under the Poincaré group

The modern description of the particle physics is through the quantum field theory. There, classical fields have a quantum version. Particles are understood as excitations above the vacuum state $|0\rangle$ of the field, which is the one with the minimum energy possible. To construct this machinery, it is necessary to regard that the fields are necessarily defined in a space-time with some kind of symmetries (or perhaps none). Internal symmetries of the fields are also relevant, because they lead to a description of the interaction processes that are observed in nature. This is important, since the dynamical features of the fields are determined by an action functional of the Lagrangian density, which can be constructed by taking into account symmetry preservation. In this chapter I will focus only in the Minskowski space-time, symmetric under transformations of the Poincaré group. Internal and space-time symmetries can be embraced by the Lie-groups.

2.1 Lie algebras

A Lie group G is a group whose elements $g \in G$ depend softly on a set of real parameters $\{\alpha_a\}_{a=1,..,N}$. The identity of the group may be conveniently defined at $\alpha_a = 0$, [65]

$$g(\alpha_a)|_{\alpha_a=0} := e. \tag{2.1}$$

Lie groups can be linearly represented as $D(g(\alpha_a)) \equiv D(\alpha_a)$, where elements can be shown as matrices that act on a vector space. The identity has a representation as well:

$$D(\alpha_a)|_{\alpha_a=0} = \mathbb{I} \tag{2.2}$$

The two reresentations are *equivalent*, if the vector bases can be related by a linear transformation. If there is not such a relation, the representations are *inequivalent*. Considering for now and without loss of generality the dependence on only one real parameter α, if a vicinity $D(d\alpha)$ of the identity is considered, a Taylor expansion can be made:

$$D(d\alpha) = 1 + id\alpha_a X_a + ..., \tag{2.3}$$

with $X_a := -i\frac{\partial}{\partial\alpha}D(\alpha)|_{\alpha=0}$ the *generators* of the group representation. The imaginary elements i are put by hand just to get Hermitian generators when the representation is

unitary[1]. The number of generators is equal to the number of real parameters α_a. Considering that multiple infinitesimal transformations can correspond to a finite transformation, an exponential form is implied:

$$D(\alpha) = \lim_{k\to\infty} \left(1 + i\frac{\alpha_0}{k} X_a\right)^k = e^{i\alpha X_a}. \tag{2.4}$$

However, the generators X_a have some restrictions for the representation to share the multiplication properties of a group. That is, in the exponential form:

$$e^{i\alpha_a X_a} e^{i\beta_b X_b} = e^{i\delta_a X_a}, \tag{2.5}$$

for some real set δ_a. By Taylor expanding and comparing the terms, the following relation for δ_a is seen:

$$[\alpha_a X_a, \beta_b X_b] = \alpha_a X_a \beta_b X_b - \beta_b X_b \alpha_a X_a = -2i(\delta_c - \alpha_c - \beta_c) X_c \tag{2.6}$$

The sets of real parameters α_a, β_b and δ_c can be redefined into a single one f_{abc}, called the *structure constants*. In this notation is written the *Lie algebra* in its usual form:

$$[X_a, X_b] = i f_{abc} X_c \tag{2.7}$$

The Lie algebra satisfies the Jacobi identity,

$$[X_a, [X_b, X_c]] + [X_c, [X_a, X_b]] + [X_b, [X_c, X_a]] = 0, \tag{2.8}$$

and it can be also said that algebras that satisfy the Jacobi identity are Lie algebras. Structure constants can form also a representation of the Lie algebra. Explicitly, if one defines $[T_a]_{bc} := i f_{abc}$, then:

$$[T_a, T_b] = i f_{abc} T_c \tag{2.9}$$

This is called the *adjoint representation.*

2.2 Poincaré group

A model in particle physics should be consistent with special relativity in a local framework. This means that the physics should be invariant under transformations between inertial frames in the Minkowski space. Such transformation can be rotations, translations, time shifts and Lorentz boosts. In other words, the Poincaré group is the direct sum between the Lorentz group and the group of spatiotemporal translations. From this moment the system of natural units will be adopted, i.e $\hbar = c = 1$. The Minkoswki's

[1] An Hermitian generator X_a satisfies that $X_a^\dagger = X_a$

metric is given by;

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \tag{2.10}$$

where $\eta_{\mu\nu}$ is the metric tensor in the $(-,+,+,+)$ signature. All transformations of space-time coordinates x^μ that leave 2.10 invariant are from the $O(3,1)$ group. The Lorentz group $SO(3,1)$ is a sub-group from $O(3,1)$, where the transformation matrices Λ are restricted by $det(\Lambda) = 1$. Moreover, continuous symmetries are also a restricted Lorentz group $L_+^\uparrow$ where $\lambda_0^0 \geq 1$ [66]. Coordinates transform under the Lorentz group as:

$$x'^\mu = \Lambda^\mu_\nu x^\nu. \tag{2.11}$$

Generators of the group are found by considering infinitesimal transformations,

$$\Lambda^\mu_\nu = \delta^\mu_\nu + \omega^\mu_\nu, \tag{2.12}$$

and thanks to the invariance of the Minkowski's metric (2.10), the elements $\omega_{\mu\nu}$ are antisymmetric, i.e

$$\omega_{\mu\nu} = -\omega_{\nu\mu}. \tag{2.13}$$

Thereby, there are 6 independent parameters. The six independent elements $\omega_{\mu\nu}$ are related to six independent generators that we can denote as $J_{\mu\nu}$, so that $J_{\mu\nu} = -J_{\nu\mu}$. A generic element of the Lorentz group can be expressed in the exponential form:

$$\Lambda = e^{\frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}}. \tag{2.14}$$

Taking a representation R with a base ϕ^i, for $i = 1, ..., n$, the action of a Lorentz transformation reads

$$\phi^i \to \left[e^{\frac{i}{2}\omega_{\mu\nu}J_R^{\mu\nu}}\right]^i_j \phi^j. \tag{2.15}$$

The physical representations can be classified according to the form of the equation (2.15). A scalar is a quantity that remains invariant under a Lorentz transformation. A vector V^μ transforms in the same way as the coordinates x^μ,

$$V^\mu \to \Lambda^\mu_\nu V^\nu. \tag{2.16}$$

When the Lorentz index μ is a superscript, the vector representation is called *covariant*. *Contravariant* vectors are defined as $V_\mu = \eta_{\mu\nu}V^\nu$, and they transform inversely compared to the covariant vectors,

$$V_\mu \to (\Lambda^\mu_\nu)^{-1} V_\nu = \Lambda^\nu_\mu V_\nu. \tag{2.17}$$

Thus, any contraction of a covariant vector with a contravariant one gives a scalar representation. The vector representation is non-trivial and therefore it is useful for finding the corresponding algebra of the Lorentz group. The generators $J^{\mu\nu}$ in the vector representation are 4×4 matrices, and they are given by:

$$(J^{\mu\nu})^\rho_\sigma = i(\eta^{\mu\rho}\delta^\nu_\sigma - \eta^{\nu\rho}\delta^\mu_\sigma), \tag{2.18}$$

The generators satisfy a Lie algebra:

$$[J^{\mu\nu}, J^{\rho\sigma}] = i(\eta^{\nu\rho}J^{\mu\sigma} - \eta^{\mu\rho}J^{\nu\sigma} - \eta^{\mu\sigma}J^{\nu\rho} + \eta^{\mu\sigma}J^{\nu\rho}). \tag{2.19}$$

Rearranging the six independent components $J^{\mu\nu}$ in terms of two spatial vectors, which are the rotation and Lorentz boost generators:

$$J^i \equiv \frac{1}{2}\epsilon^{ijk}J^{jk}, \qquad K^i \equiv J^{i0}, \tag{2.20}$$

the following Lie algebra is derived:

$$[J^i, J^j] = i\epsilon^{ijk}J^k, \tag{2.21}$$
$$[J^i, K^j] = i\epsilon^{ijk}K^k, \tag{2.22}$$
$$[K^i, K^j] = -i\epsilon^{ijk}J^k. \tag{2.23}$$

This is the same form of a $SU(2)$ group's algebra, thus the J^i can represent angular momentum. Redefining the parameters $\omega^{\mu\nu}$ as

$$\theta^i \equiv \frac{1}{2}\epsilon^{ijk}\omega^{jk}, \qquad \eta^i \equiv \omega^{i0}, \tag{2.24}$$

the equation (2.14) turns out to be:

$$\Lambda = e^{-i\theta^i J^i + i\eta^i K^i}. \tag{2.25}$$

The parameters θ^i are the three angles of the spatial rotations and the remaining parameters, η^i, are the three *rapidities*, which are related to the relative velocity v^i between two inertial frames by $v^i = \tanh\eta^i$. On irreducible representations, the operator $J^2 = J^iJ^i$ is the *Casimir* of the angular momentum algebra (2.21), and takes the form $j(j+1)\mathbb{I}$, with $j = 0, 1/2, 1, 3/2....$ If one represents a particle state of spin j, the dimension of the representation is $2j+1$.

2.2.1 Spinor representation

A spinor representation have two inequivalent parts labeled as (j_-, j_+), where the dimension is $(2j_- + 1)(2j_+ + 1)$. This labeling is possible because when the following operators are defined

$$J^{\pm,i} = \frac{J^i \pm iK^i}{2}, \tag{2.26}$$

they fulfill idependent algebras:

$$[J^{+,i}, J^{+,j}] = i\epsilon^{ijk}J^{+,k}, \tag{2.27}$$
$$[J^{-,i}, J^{-,j}] = i\epsilon^{ijk}J^{-,k}, \tag{2.28}$$
$$[J^{+,i}, J^{-,j}] = 0. \tag{2.29}$$

The spinors belonging to the $(1/2, 0)$ representation are named left-handed Weyl spinors ψ_L and the other ones are called right-handed Weyl spinors ψ_R. In this section the spinor indices are not going to be explicit. The generators of such representations are 2×2

matrices $\frac{\sigma^i}{2}$, for $i = 1, 2, 3$; where σ^i are the Pauli Matrices and they are given by:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.30}$$

In the left-handed weyl spinor representation, the generators read as $J^- = \frac{\sigma^i}{2}$ and $J^+ = 0$, and for the right-handed ones the corresponding generators are $J^+ = \frac{\sigma^i}{2}$ and $J^- = 0$. Therefore, using the equations (2.26) and (2.25), the Weyl spinors transform in the following way under a Lorentz transformation:

$$\begin{aligned} \Lambda_L \psi_L &= e^{(-\theta^i - \eta^i)\frac{\sigma^i}{2}} \psi_L, \\ \Lambda_R \psi_R &= e^{(-\theta^i + \eta^i)\frac{\sigma^i}{2}} \psi_R. \end{aligned} \tag{2.31}$$

Defining the charge conjugation of a Weyl Spinor as

$$\psi_L^C = i\sigma^2 \psi_L^*, \qquad \psi_R^C = -i\sigma^2 \psi_R^*, \tag{2.32}$$

it can be shown that ψ_R^C belongs to the $(1/2, 0)$ representation and, similarly, ψ_L^C belongs to the $(0, 1/2)$ representation. It can be also seen that $\left(\psi_{R/L}^C\right)^C = \psi_{R/L}$.

The Dirac spinors Ψ belong to the $(1/2, 1/2)$ representation, and in the so called chiral basis, they are arrangements conformed of left-handed and right-handed spinors in the next form

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{2.33}$$

Thus, the action of a Lorentz transformation on a Dirac spinor reads:

$$\Lambda_D \Psi = \begin{pmatrix} \Lambda_L & 0 \\ 0 & \Lambda_R \end{pmatrix} \Psi. \tag{2.34}$$

According to the properties of the charge conjugation, such an operation on a Dirac spinor reads

$$\Psi^C = \begin{pmatrix} \psi_R^C \\ \psi_L^C \end{pmatrix} \tag{2.35}$$

The last of the spinors that are going to be introduced are the *Majorana spinors*. Those quantities are Dirac spinors, but in the following form

$$\Psi_M = \begin{pmatrix} \psi_L \\ \psi_L^C \end{pmatrix}. \tag{2.36}$$

This means that $\Psi_M^C = \Psi_M$, and also that the Majorana Spinors have two degrees of freedom, regardless of having four components.

2.3 Field representations of the Poincaré group

The basic physical representations of the Poincaré group had been defined. With that as a background, it is possible to describe physical fields in the Minkowski space as field representations of the Poincaré group. A classical field is a quantity defined over the coordinates of some manifold. What concerns the present work is the Minkoswki space-time. The classical evolution of the fields is governed by the minimization of the action functional over a Lagrangian density $\mathcal{L}$:

$$S = \int dx^4 \mathcal{L}(\phi_\alpha, \partial_\mu \phi_\alpha, x^\mu), \tag{2.37}$$

where ϕ_a represents a set of fields. The minimization condition $\delta S = 0$, together with fixed boundary conditions, leads to the dynamical equations of the field:

$$\frac{\partial L}{\partial \phi_\alpha} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_\alpha)} \right). \tag{2.38}$$

Knowing how a field transforms under a symmetry group is fundamental, since the Lagrangian density is an addition of all invariant terms that can be constructed from the field. However, the set of renormalizable Lagrangians is much smaller. Renormalizable Lagrangians are the ones that are useful for physical predictions, since no infinite counterterms are needed.

2.3.1 Scalar fields

The scalar field representation of the Poincaré group is invariant under the group transformations, $\phi'(x') = \phi(x)$. Under infinitesimal transformations $x'^\mu = x^\mu + \delta x^\mu$, the scalar field fulfills at a fixed point that:

$$\phi'(x) = \phi(x) - \frac{i}{2} \omega_{\mu\nu} L^{\mu\nu} \phi(x), \tag{2.39}$$

where $L^{\mu\nu} = i(x^\mu \partial^\nu - x^\nu \partial^\mu)$ is the angular momentum operator. Therefore, this field has no intrinsic angular momentum, being suitable for describing spin-0 particle states. A complex scalar field $\phi(x)$ can be described by two real scalar fields as $\phi(x) \equiv \phi_1(x) + i\phi_2(x)$. This is convenient to review, since the field which describes the Higgs field in the Standard Model requires complex scalar fields. The Lagrangian that corresponds to a scalar complex field ϕ and whose particles have a rest mass m is given by:

$$\mathcal{L}_{KG} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi. \tag{2.40}$$

The first term is kinetic and the second one is related to the rest mass m. The equations 2.38 imply the Klein-Gordon equation:

$$(\partial_\mu \partial^\mu + m^2)\psi = 0 \tag{2.41}$$

This Lagrangian has a $U(1)$-symmetry since it is invariant under the phase transformations $\phi(x) \to e^{i\theta}\phi(x)$. The conserved current is the following one,

$$j_\mu = -i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi). \tag{2.42}$$

This current couples with the electromagnetic field. Thereby, if a scalar field is complex, it can have an electric charge. This is not possible in real scalar fields, because (2.42) vanishes identically. Charged scalar fields coupled to charged gauge bosons can explain the mass of interactive gauge bosons like $W^\pm$. Moreover, neutral complex scalar fields are also important for giving an origin to the mass of the Z boson.

2.3.2 Spinor fields

In the Weyl representation, left-handed and right-handed spinors transform respectively under the Poincaré group as:

$$\begin{aligned} \psi'_L(x') &= \Lambda_L\psi_L(x), \\ \psi'_R(x') &= \Lambda_R\psi_R(x). \end{aligned} \tag{2.43}$$

At a fixed point, the spin of the field can be determined by an infinitesimal transformation:

$$\psi'_{L/R}(x) = \psi_{L/R}(x) - \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu}\psi_{L/R}(x), \tag{2.44}$$

where $J^{\mu\nu} = L^{\mu\nu} + S^{\mu\nu}$ is the total angular momentum, involving the orbital and the spin. The $S^{\mu\nu}$ are given by the equation (2.26). Therefore, these fields can describe spin-1/2 particles, like quarks and leptons. The invariant Lagrangian density under the Poincaré group of Weyl spinor fields with rest mass m is given by:

$$\mathcal{L}_D = i\psi_L^\dagger\bar{\sigma}^\mu\partial_\mu\psi_L + i\psi_R^\dagger\sigma^\mu\partial_\mu\psi_R - m(\psi_L^\dagger\psi_R - \psi_R^\dagger\psi_L), \tag{2.45}$$

where the two first terms are kinetic and the last ones are involved with the rest mass. Important to note here is that both chiralities, right and left are needed for making possible the rest mass of fermions in a relativistic model. The Lagrangian density (2.45) implies the Dirac equation, written in the Weyl form:

$$\begin{aligned} \bar{\sigma}^\mu i\partial_\mu\psi_L &= m\psi_R \\ \sigma^\mu i\partial_\mu\psi_R &= m\psi_L \end{aligned} \tag{2.46}$$

Involving the two chiralites ψ_R and ψ_L, a Dirac spinor Ψ like in the equation 2.34 follows the Dirac equation in the usual form:

$$(i\gamma^\mu\partial_\mu - m)\Psi = 0, \tag{2.47}$$

where γ^μ are the Dirac matrices that fulfill the Clifford algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{2.48}$$

Chiral states will remain the same if the mass is null. In this situation, there is a chiral symmetry. In other words, it means that the Lagrangian (2.45) is invariant under the transformation $\Psi \to e^{i\alpha\gamma^5}\Psi$, with α a real parameter and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The implied conserved current is an axial vector given by (2.49), whose conservation law reads $\partial_\mu J^{\mu 5}$:

$$j^\mu_A = \bar{\Psi}\gamma^\mu\gamma^5\Psi \tag{2.49}$$

However, when the quantization of the fields is made, anomalies arise in gauge theories. A model should avoid anomalies, since they affect the renormalizability of the theory. Chiral anomalies will be covered within the chapter (3).

2.3.3 Vector fields

Covariant vector fields transform under the Poincaré group as covariant vectors do:

$$V'^\mu(x') = \Lambda^\mu{}_\nu V(x) \tag{2.50}$$

A similar equation holds for contravariant vectors, but with an inverse of the transformation that is done to the covariant counterpart. Vectors can be seen as a $(\frac{1}{2},\frac{1}{2})$ representations of the Lorentz group. Thus, there is a way to express a vector in terms of Weyl spinors. For example, the following terms behave as contravariant vectors:

$$\chi^\dagger_R\sigma^\mu\psi_R \qquad\qquad \chi^\dagger_L\bar{\sigma^\mu}\psi_L \tag{2.51}$$

The dynamics of the fields will be determined in our dicussion in the case of minimal coupling with gauge fields.

2.4 Minimal gauge coupling

Interaction of vector fields with matter can be regarded with a minimal coupling. In nature, particle states seem arranged under doublets of $SU(2)_L$ for left-handed leptons in weak interaction and triplets of $SU(3)_C$ for quarks in strong interactions. Let us regard a $SU(N)$ multiplet that transforms like $\psi^j \to U_{jk}\psi^k$, with U_{jk} an unitary matrix from $SU(N)$. The Lagrangian density with minimal coupling of vector fields $A_\mu(x)$ with matter is similar to (2.45):

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu + igA_\mu)\psi, \tag{2.52}$$

where g is a coupling constant. This Lagrangian density is invariant under the local transformation $U = e^{ig\alpha^a(x)T^a}$, with T^a the generators of SU(N), if the vector fields A_μ transform like:

$$A'_\mu = UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.53}$$

This is a similar form to the gauge invariance in quantum electrodynamics (QED). Thereby, interactions can be introduced by considering an invariant Lagrangian density under local transformation. The Lagrangian (2.52) does not contain a mass term, since in general, it is not invariant under gauge transformations. The equation (2.53) is consistent if the A_μ is written like:

$$A_\mu = T^a A^a_\mu. \tag{2.54}$$

This means that there are as much of the interaction fields as the number of generators of the symmetry considered. The recipe to include a minimal coupling is done by a geometrical interpretation; the derivative is changed by a covariant derivative $\partial_\mu \to D_\mu = \partial_\mu + igA_\mu$, so the Lagrangian (2.52) is written as:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi = i\bar{\psi}\not{D}\psi, \tag{2.55}$$

where the slash notation $\gamma^\mu A_\mu \equiv \not{A}$ is adopted from now on. Kinetic terms of this field are built in analogy to the QED, where one first defines an antisymmetric range 2 tensor,

$$F_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu], \tag{2.56}$$

where F_μ corresponds to the electromagnetic tensor in the abelian case, $[A_\mu, A_\nu] = 0$. Considering that the generators of the group fulfill a Lie algebra like in (2.9), and writing from (2.54) the tensor strenght fields, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a_{bc} A^b_\mu A^c_\nu$, the Lagrangian density that considers minimal coupling of vector fields with matter and kinetic terms of the vectors is given by:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}. \tag{2.57}$$

Gauge bosons are massless, since a mass term is not gauge invariant. Couplings of matter with interaction fields like in (2.57) respect the chiral symmetry classically, but generally in the quantum approach, chiral anomalies emerge.

Generators of $SU(2)$ and $SU(3)$

For $SU(2)$ there are three generators that read as $T^a = \frac{\sigma^a}{2}$, for $a = 1, 2, 3$. The σ^a are the known Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.58}$$

Therefore, field theories invariant under a local $SU(2)$ group include 3 gauge bosons. The $SU(3)$ can be generated by 3×3 hermitian matrices $T_a = \frac{1}{2}\lambda_a$, with $a = 1, 2, ..., 8$. The

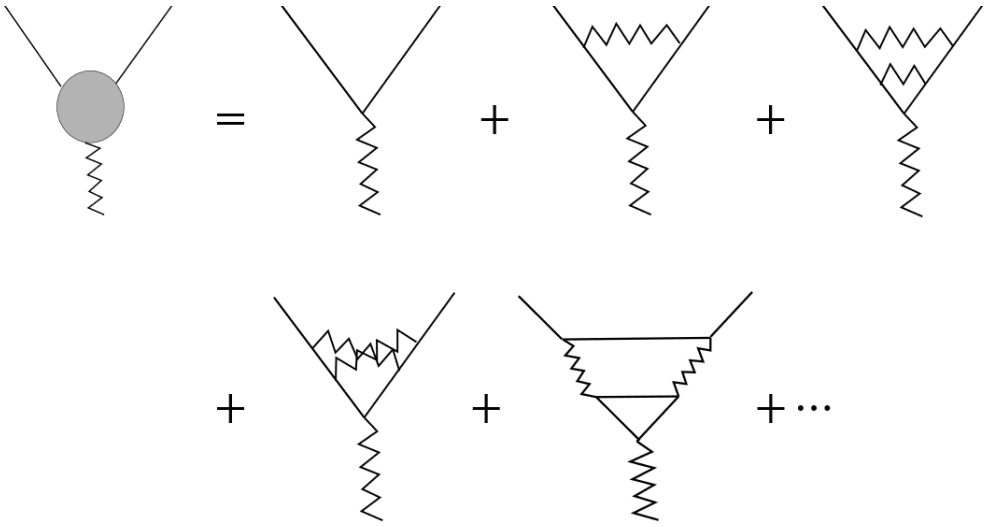

Figure 2.1: Expansion of the axial vertex function to the order of e^5.

Gell-Mann matrices λ_a are shown written as:

$$\begin{aligned}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\
\lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\end{aligned} \tag{2.59}$$

Similarly, a field theory invariant under a local $SU(3)$ group must contain 8 gauge bosons from here.

2.5 Chiral anomalies

Ward-Takahashi identities are crucial for the renormalizability of quantum field theories with symmetries, since they are relations between vertex functions and propagators that remain invariant under renormalization. Chiral anomalies can be identified when a perturbation theory is considered. They can be seen by trying to obtain a Ward-Takahashi identity of the axial current conservation, considering boson and matter field's vertices. For instance, being e the charge of the electron, the perturbation expansion of the axial vector vertex in QED is given by the figure (2.1) [67]. The last diagram, which contains triangle fermion loops, fails at satisfying the expected Ward identities. Considering the diagrams in the figure (2.2), let $T_{k\lambda\mu}$ be the contribution to the scattering amplitude of

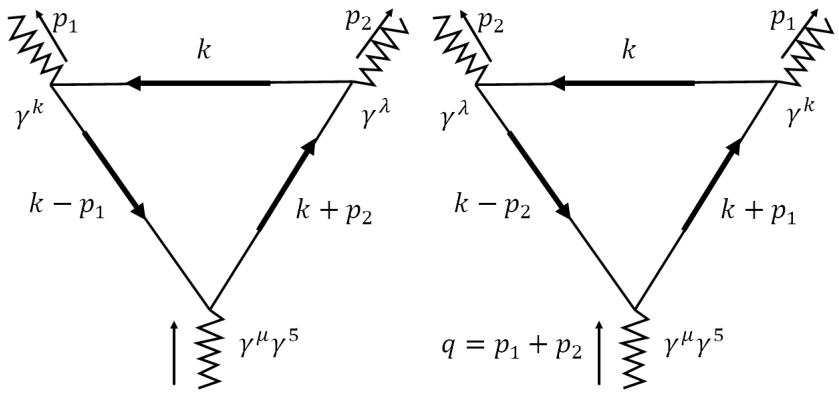

Figure 2.2: Triangle Feynman diagrams.

the fermion loop. Then, if one redefines $T_{k\lambda\mu}$ as $T'_{k\lambda\mu}$f for obtaining Ward identities for vector current conservation, it is not possible to obtain the Ward identity for axial current conservation. The identities obtained instead are:

$$\begin{aligned} p_1^k T'_{k\lambda\mu} &= 0 \quad \text{Vector currents} \\ p_2^k T'_{k\lambda\mu} &= 0 \\ (p_1+p_2)^\mu T'_{k\lambda\mu} &= \frac{1}{2\pi^2}\epsilon_{k\lambda\mu\nu}p_2^\mu p_1^\nu \neq 0. \quad \text{Axial currents} \end{aligned} \tag{2.60}$$

Since Ward identities do not hold, renormalizability of the theory is not guaranteed. This is also present in gauge theories in general. The way to avoid such problems is to cancel contributions like in diagrams (2.2) from the beginning, by considering contributions from other fermions. When building a model with gauge symmetry, the charge configuration shall be carefully chosen, since it is the key to cancel chiral anomalies.

Chapter 3

The Standard Model and $U(1)_X$ extensions

3.1 The Standard Model

There is no doubt among particle physicists that the Standard Model has been very successful when being contrasted with experimental data. It is quantum field theory described under the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. $SU(3)_C$ involves 8 massless gluons for strong interactions and $SU(2)_L \times U(1)_Y$ the massive bosons W and Z and the massless photon A. The model contains three families of fermions, whose generations are only distinguished by the mass of the particles which it contains. For leptons, the model has the electronic, muonic and tauonic generations. The quarks also have three generations. The charge configuration of all particle fields included in the Standard Model is shown in the table (3.1), which also avoids of chiral anomalies. Since a mass term cannot be invariant under transformations of the $SU(2)_L$ group, particle masses are generated by the vacuum expectation value of a scalar doublet field called the Higgs field.

I call here the generators from $SU(3)_C$ as τ^a, with $a = 1, 2, ..., 8$. The electric charge operator is built from generators of $SU(2)_L \times U(1)_Y$. Being Y the hypercharge from $U(1)_Y$ and $T_i = \frac{\sigma_i}{2}$ the generators from $SU(2)_L$, the electric charge operator is defined in the following way:

$$Q = T_3 + \frac{Y}{2}. \tag{3.1}$$

3.1.1 Full Lagrangian of the Standard Model

The Lagrangian of the Standard Model is mainly the addition of all fermion and gauge boson Lagrangians with minimal coupling under the local $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry. Explicitly, covariant derivatives are defined in this case as:

$$D_\mu = \partial_\mu + ig_s \sum_{a=1}^{8} G_\mu^a T^a + ig \sum_{a=1}^{3} A_\mu^a \frac{\tau^a}{2} + ig' B_\mu \frac{Y}{2}, \tag{3.2}$$

where g_s, g and g' are the coupling constants from $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, respectively. Similarly, the Standard Model has the 8 gluon fields G_μ^a for strong interactions, A_μ^a are the gauge bosons of $SU(2)_L$ and B_μ the gauge boson from $U(1)_Y$. The full Lagrangian

Table 3.1: Fermion and Higgs boson arrangement in the Standard Model

Left-handed quarks	Y	T_3	Left-handed leptons	Y	T_3
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	1/3	1/2 -1/2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	-1	1/2 -1/2
$\begin{pmatrix} c \\ s \end{pmatrix}_L$	1/3	1/2 -1/2	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	-1	1/2 -1/2
$\begin{pmatrix} t \\ b \end{pmatrix}_L$	1/3	1/2 -1/2	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	-1	1/2 -1/2
Right-handed quarks	Y	T_3	Right-handed leptons	Y	T_3
u_R	4/3	0	e_R	-2	0
d_R	-2/3	0			
c_R	4/3	0	μ_R	-2	0
s_R	-2/3	0			
t_R	4/3	0	τ_R	-2	0
b_R	-2/3	0			

Higgs Boson	Y	Electric charge
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	1 0

Here Y refers to the weak hypercharge from $U(1)_Y$ and T_3 corresponds to the weak isospin from $SU(2)_L$. The subscripts L and R identify the left and right chirality, respectively.

can be expressed in four parts: One involving the quantum chromodynamics, another one the electroweak sector, the Yukawa Lagrangian for the origin of fermion masses and the Higgs potential for the main origin of all masses and spontaneous symmetry breaking of the electroweak interaction into the QED $SU(2)_L \times U(1)_Y \to U(1)_{QED}$.

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}. \tag{3.3}$$

Quantum chromodynamics affects directly only the quarks, since all other particles are $SU(3)_C$ singlets. Defining the strength tensors $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} \mathcal{A}^b_\mu G^c_\nu$ for gluons, with f^{abc} the structure constants from $SU(3)C$, the Lagrangian involving this part of the SM turns out to be:

$$\mathcal{L}_{QCD} = \sum_{j=1}^{3} i\overline{q^j}_L \not{D} q^j_L + \sum_{j=1}^{6} i\overline{u^j_R} \not{D} u^j_R + \sum_{j=1}^{3} i\overline{d^j_R} \not{D} d^j_R - \frac{1}{4} \sum_{a=1}^{8} G^a_{\mu\nu} G^{\mu\nu}_a. \tag{3.4}$$

The electroweak interactions affect all particles of the Standard Model. They are actually mediated via massive bosons Z and W, although this is not directly seen by a Lagrangian spawned by gauge symmetry, since gauge bosons are massless. This is solved by the Higgs kinetic term, but this is left for later. The Lagrangian of this sector expressed in terms

of gauge bosons is given by:

$$\begin{aligned}\mathcal{L}_{EW} = &\sum_{j=1}^{3} i\overline{q^j}_L \not{D} q^j_L + \sum_{j=1}^{3} i\overline{u^j_R} \not{D} u^j_R + \sum_{j=1}^{3} i\overline{d^j_R} \not{D} d^j_R \\ &+ \sum_{j=1}^{3} i\overline{L^j}_l \not{D} L^j_L + \sum_{j=1}^{3} i\overline{e^j_R} \not{D} e^j_R \\ &+ (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ &- \frac{1}{4}\sum_{a=1}^{3} A^a_{\mu\nu} A^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \end{aligned} \tag{3.5}$$

The understanding of the mass origin of the masses relies in the Higgs potential given in (3.6). It will be shown more in detail in this chapter, but the potential has a form with multiple minima. Therefore, for having an unique vacuum, the symmetry must be broken in the vacuum state.

$$\mathcal{L}_{Higgs} = -\mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 = -V(\Phi), \tag{3.6}$$

Mass terms like in the Dirac Lagrangian (2.45), $m(\psi_R \psi_L + \psi_L \psi_R)$, are not invariant under the local $SU(2)_L$. This is simply because left-handed fermions are doublets under $SU(2)_L$ and the right-handed are singlets. The Yukawa Lagrangian is thus considered in the Standard Model as trilinear terms between the Higgs doublet and the fermions. Both fermion chiralities are required to build up such trilinear terms that are invariant under $SU(2)_L \times U(1)_Y$. After SSB, trilinear terms will effectively be bilinear ones, which is interpreted as mass terms. The Yukawa Lagrangian is given by:

$$\mathcal{L}_{Yukawa} = -Y^l_{jk} \overline{L^j}_L \Phi e^k_R - Y^u_{jk} \overline{q^j}_L \tilde{\Phi} u^k_R - Y^d_{jk} \overline{q^j}_L \Phi d^k_R + H.c, \tag{3.7}$$

where H.c means hermitic conjugate and the Y's are the Yukawa coupling constants. The conjugate $\tilde{\Phi} \equiv = iT_2 \Phi^*$ couples to up-like quarks and would couple with neutrinos if right-handed chiralities have existed in the Standard Model. But this is not the case, so neutrinos in the Standard Model are massless.

3.1.2 The Higgs mechanism

The Higgs doublet consists of two complex scalar fields, one charged and one neutral. Therefore, it has 4 real scalar fields in total,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{3.8}$$

Defining from the potential (3.6) $v \equiv \sqrt{-\frac{\mu^2}{\lambda}}$, it can be shown that up to an irrelevant constant the Higgs potential can be written as:

$$V(\Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right). \tag{3.9}$$

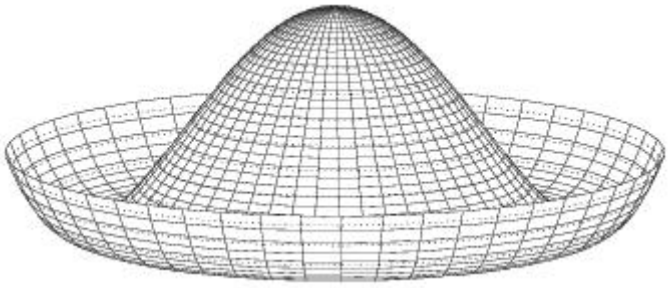

Figure 3.1: Mexican hat potential. Taken from `https://physicsmasterclasses.org/exercises/hands-on-cern/hoc_v21en/main_frame/sm_higgs2.html`.

Therefore, the potential has a minimum that can be geometrically identified with a 3-sphere of radius $\frac{v^2}{2}$, because $\Phi^\dagger\Phi = (\phi_1)^2 + (\phi_2)^2 + (\phi_3)^2 + (\phi_4)^2 = \frac{v^2}{2}$. v is called the vacuum expectation value (VEV) of the Higgs field. A widely used schematic view of the potential is a two-dimensional cut of it, well known as the Mexican hat potential shown in figure (3.1) [68]. For ensuring the existence of the minima region, $\lambda > 0$. The spontaneous symmetry breaking happens when choosing an unique vacuum expectation state that is electrically neutral and it reads:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{3.10}$$

This vacuum expected doublet is not invariant under $SU(2)_L \times U(1)_Y$ but is invariant under $U(1)_Q$. Thereby, this vacuum state breaks the symmetry spontaneously, as it can be seen by the action of the group generators on it in equation (3.11).

$$\begin{aligned} T_1\langle \Phi \rangle &= \frac{1}{2\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, & T_2\langle \Phi \rangle &= \frac{i}{2\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, \\ T_3\langle \Phi \rangle &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, & Y\langle \Phi \rangle &= \langle \Phi \rangle. \end{aligned} \tag{3.11}$$

Thanks to the Goldstone theorem [69], for every generator broken, there will be a Goldstone boson. They come from longitudinal perturbations of the vacuum and therefore they are massless. They become longitudinal components of the bosons Z and W, which is interpreted as the mass. Since $U(1)_Q$ is the unbroken symmetry, the photon remains massless. Using the local symmetry $SU(2)_L$, the Higgs doublet (3.8) can be rewritten in the *unitary gauge*,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \tag{3.12}$$

Excitations over the vacuum are regarded by the Higgs field $h(x)$. The Higgs potential becomes in the unitary gauge:

$$V_H = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4. \tag{3.13}$$

A mass term $\lambda v^2 H^2$ is present. Therefore, the bare mass of the Higgs boson is:

$$m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}, \tag{3.14}$$

Boson masses

Gauge boson masses come from the kinetic terms of the Higgs doublet with minimal coupling. Explicitly in the unitary gauge, the kinetic term turns out to be:

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2v^2}{8}\left[(A_1^\mu)^2 + (A_2^\mu)^2\right] + \frac{v^2}{8}(gA_3^\mu - g'B_\mu)^2 + ..., \tag{3.15}$$

where cubic and quartic terms are not explicitly shown since they are not relevant at this moment. Mass terms for the boson appear thanks to the VEV. The fields A_1^μ and A_2^μ get a bare mass of $\frac{gv}{2}$, but the electroweak interactions with charged bosons are described by $W^\mu = \frac{A_1^\mu - iA_2^\mu}{\sqrt{2}}$, which are also mass eigenstates. The equation (3.15) suggests the definition of two orthogonal fields Z^μ and A^μ for the Z bosons and the photon, respectively. They are defined by:

$$\begin{aligned} Z^\mu &= \frac{gA_3^\mu - g'B^\mu}{(g^2+g'^2)^{\frac{1}{2}}} \equiv \cos\theta_W A_\mu^3 - \sin\theta_W B^\mu, \\ A^\mu &= \frac{g'A_3^\mu + gB^\mu}{(g^2+g'^2)^{\frac{1}{2}}} \equiv \sin\theta_W A_\mu^3 + \cos\theta_W B^\mu, \end{aligned} \tag{3.16}$$

where $\sin^2\theta_W \approx 0.24$. θ_W is the Weinberg angle, defined by:

$$\cos\theta_W = \frac{g}{((g^2+g'^2)^{\frac{1}{2}})} \qquad\qquad \tan\theta_W = \frac{g'}{g} \tag{3.17}$$

The coupling of the Higgs doublet with the gauge bosons explains the origin of the masses of bosons. They are in the SM:

$$\begin{aligned} m_A &= 0 \\ m_W &= \frac{gv}{\sqrt{2}} \\ m_Z &= \frac{M_W}{\cos\theta_W}. \end{aligned} \tag{3.18}$$

For neutral gauge bosons, the mass eigenstates are different from the gauge states. They can be related by a rotation with the angle θ_W:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W \\ \cos\theta_W & -\sin\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \tag{3.19}$$

This mixing indicates that the neutral Z boson and the photon A interact with the charged W, thanks to the kinetic terms of gauge bosons. The Feynman rule for a trilinear interaction between the Z and W bosons is given in the figure (3.2) [70].

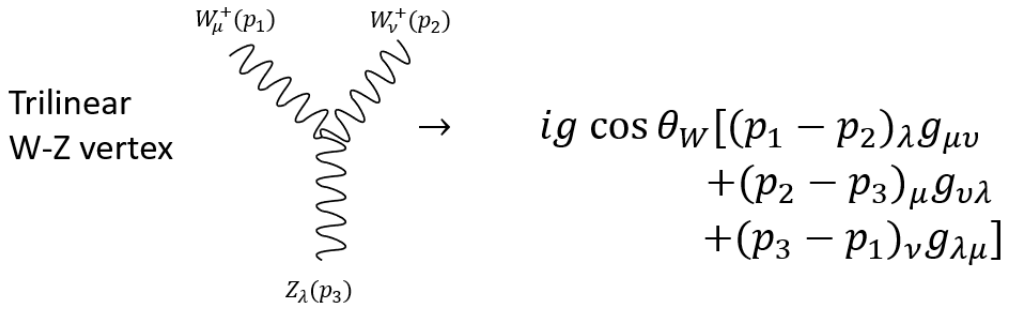

Figure 3.2: Trilinear Vertex Feynman rule for electroweak bosons.

Fermion masses

Fermion masses are explained by the Yukawa Lagrangian (3.7), thanks to the SSB as well. In the unitary gauge for the Higgs doublet, the leptonic Yukawa Lagrangian reads

$$\mathcal{L}_{H,L} = -\left(\frac{v+H}{\sqrt{2}}\right) Y^l_{\alpha\beta}\overline{L_{\alpha L}} e_{\beta R} + H.c, \tag{3.20}$$

The coupling constants $Y^l_{\alpha\beta}$ can be arranged in a matrix which is not generally diagonal. In this matrix representation, lepton flavor can also be arranged in the following way:

$$l_L = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \quad \text{and} \quad l_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}. \tag{3.21}$$

leaving the leptonic Yukawa Lagrangian in a matrix form,

$$\mathcal{L}_{H,L} = -\left(\frac{v+H}{\sqrt{2}}\right) Y^l_{\alpha\beta}\overline{l_L} Y^l l_R + H.c. \tag{3.22}$$

The diagonalization can be performed by a biunitary transformation on the arrangements (3.21)

$$l'_L = V_L^{l\dagger} \equiv \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad \text{and} \quad l'_R = V_R^{l\dagger} \equiv \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}; \tag{3.23}$$

the primed basis is called the *mass basis*, whereas the non-primed basis is known as the *flavor basis*. The transformations V^l_L and V^l_R diagonalize the Yukawa coupling matrices:

$$V_L^{l\dagger} Y^l V_R^l = Y'^l, \tag{3.24}$$

with $Y'^l_{\alpha\beta} = y^l_\alpha \delta_{\alpha\beta}$ diagonal. Thus, the Yukawa Lagrangian can be written in a diagonal form:

$$\mathcal{L}_{H,L} = -\frac{y^l_\alpha v}{\sqrt{2}}\overline{l_\alpha} l_\alpha - \frac{y^l_\alpha}{\sqrt{2}}\overline{l_\alpha} l_\alpha H. \tag{3.25}$$

The masses of the electrically charged leptons are shown in equation (3.26). Of course, since the SM does not count on right-handed neutrinos, such neutral particles remain

massless in the theory.

$$m_\alpha = \frac{y_\alpha^l v}{\sqrt{2}} \quad (\alpha = e, \mu, , \nu). \tag{3.26}$$

In the case of quarks, their masses can be obtained in an analogous way as is was shown for leptons. The idea is to first define the following arrangements for having a matrix form of the Yukawa Lagrangian of quarks:

$$U_L = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \quad , \quad U_R = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \quad D_L = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \quad \text{and} \quad D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \tag{3.27}$$

Similarly, the Yukawa Lagrangian can be left diagonal if biunitary transformations are performed on the arrangements:

$$U_L' = V_L^{\dagger U} U_L \quad , \quad U_R' = V_R^{\dagger U} U_R, \quad D_L' = V_L^{\dagger D} D_L \quad \text{and} \quad D_R' = V_R^{\dagger D} D_R \tag{3.28}$$

Therefore, following the prescription of the equation (3.24), the masses of quarks are:

$$m_\alpha = \frac{y_\alpha^U v}{\sqrt{2}} \quad (\alpha = u, c, t), \tag{3.29}$$

$$m_\beta = \frac{y_\beta^D v}{\sqrt{2}} \quad (\beta = d, s, b), \tag{3.30}$$

with y_α^U and y_α^D the Yukawa coupling constants in the mass basis for the up-like quarks and the down-like quarks, respectively. All fermion masses depend on two ingredients: A VEV of a scalar field and Yukawa coupling constants. It was not possible for the SM to predict all phenomenology involved with fermion masses, because the Yukawa couplings are almost free parameters. Specially, how come that the fermion masses are so different amongst each other having a priori only one energy scale v? The answer from SM is a fine-tuning of the Yukawa parameters, and this is a naturalness problem called *the fermion mass hierarchy problem.*

3.1.3 Electroweak interactions

The Lagrangian in (3.5) is not properly written in terms of the massive boson states. It is important to have them explicitly, since those states are the ones that are really observed. Two types of interactions arise in the electroweak sector: Charged currents, mediated by $W^\pm$ bosons, and neutral currents, mediated by the photon A and the Z boson. Although the topic of Feyman rules is not reviewed in this thesis, I will simply show the rules for vertices. Let us see first the case for leptons, since in the Standard Model the electroweak interactions can be diagonal in the leptonic mass eigenstates. This is because transformations between flavor and mass eigenstates like in (3.24) leave invariant the Electroweak Lagrangian for leptons. This would be not the case if right-handed neutrinos existed, since an additional transformation for leaving the Yukawa Lagrangian has to be done, which is in general different from (3.24). In such case, the electroweak interaction would not be diagonal in the mass basis. The charged current Lagrangian for leptons

turns out to be,

$$\mathcal{L}^{(CC)}_{EW,I,Lep} = -\frac{g}{2\sqrt{2}} j^{\mu}_{W,L} W_{\mu} + H.c, \tag{3.31}$$

where $j^{\mu}_{W,L}$ is the leptonic charged current for the three generations, given by:

$$j^{\mu}_{W,L} = 2 \sum_{f=e,\mu,\tau} \overline{\nu_{fL}} \gamma^{\mu} f_L. \tag{3.32}$$

In the case of neutral currents for leptons, the Lagrangian reads

$$\begin{aligned} \mathcal{L}^{(NC)}_{EW,I,Lep} = &- \frac{g}{2\cos\theta_W} \sum_{f=e,\mu,\tau} [\overline{\nu_{fL}} \not{Z} \nu_{fL} - (1 - 2\sin^2\theta_W) \overline{f_L} \not{Z} f_L + 2\sin^2\theta_W \overline{f_R} \not{Z} f_R] \\ &+ g \sin\theta_W \sum_{f=e,\mu,\tau} \overline{e} \not{A} e. \end{aligned} \tag{3.33}$$

The last term in (3.33) should correspond with the QED Lagrangian. Thereby, being q the charge of the electron,

$$g \sin\theta_W = q \tag{3.34}$$

Therefore, the neutral current associated with the photon is just the electromagnetic fermion current:

$$j^{\mu}_{A,L} = - \sum_{f=e,\mu,\tau} \overline{f} \gamma^{\mu} f. \tag{3.35}$$

For interactions with the Z boson, it is possible to write the Lagrangian of lepton interactions with this boson:

$$\mathcal{L}^{(Z)}_{I,L} = -\frac{g}{2\cos\theta_W} j^{\mu}_{Z,L} Z_{\mu}. \tag{3.36}$$

Introducing the coefficients $g^f_L = T^f_3 \sin^2\theta_W$ and $g^f_R = -q_f \sin^2\theta_W$ for the fermion field f with electric charge q_f, the leptonic neutral current is:

$$j^{\mu}_{Z,L} = \sum_{f=e,\mu,\tau} \left[2 g^{\nu_f}_L \overline{f_L} \gamma^{\mu} \nu_{fL} + 2 g^f_L \overline{f_L} \gamma^{\mu} \nu_{fL} + 2 g^f_R \overline{f_R} \gamma^{\mu} f_R \right]. \tag{3.37}$$

Additionally, vector and axial coupling constants are defined as $g^f_V = g^f_L + g^f_R$ and $g^f_A = g^f_L - g^f_R$. From the fermions currents shown so far, the Feynman diagrams and vertex rules are shown in the figure (3.3). In the case of quarks, neutral currents happen to have the same form. However, in charged currents the Yukawa Lagrangian has strong implications.

For quarks it can be demonstrated that the charged-current Lagrangian reads

$$\mathcal{L}^{(CC)}_{I,Q} = -\frac{g}{2\sqrt{2}} j^{\mu}_{W,Q} W_{\mu} + H.c, \tag{3.38}$$

where the quark charged current is:

$$j^{\mu}_{W,Q} = 2 \sum_{\text{3 generations}} \overline{u_L} \gamma^{\mu} d_L, \tag{3.39}$$

However, this is on the flavor basis. Transforming to the mass basis using equations 3.28,

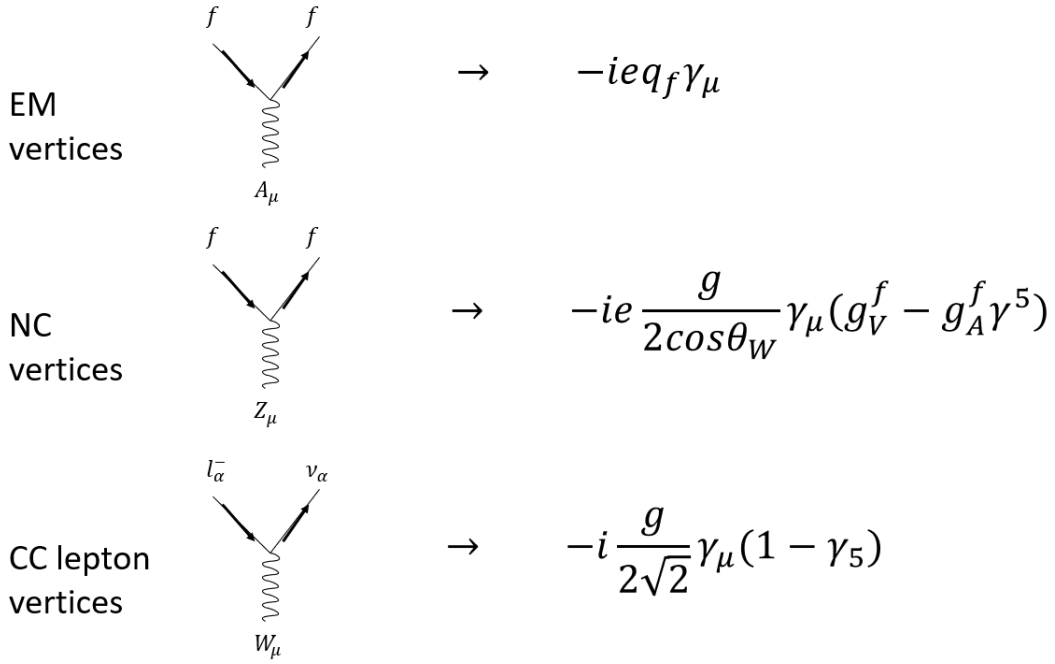

Figure 3.3: Vertex Feynman rules for leptons

the quark charge current shall be written as:

$$j^\mu_{W,Q} = 2\overline{U}_L\gamma^\mu V D_L, \tag{3.40}$$

where $V = V_L^{U\dagger}V_L^D$ corresponds to the Cabbibo-Kobayashi-Maskawa (CKM) matrix, and it accounts for flavor changes in charged current interactions.

The neutral currents are for quarks:

$$\mathcal{L}^{(NC)}_{I,Q} = -qj^\mu_{\gamma,Q}A_\mu - \frac{g}{2\cos\mathcal{V}_W}j^\mu_{Z,Q}Z_\mu, \tag{3.41}$$

where $j^\mu_{\gamma,Q}$ and $j^\mu_{Z,Q}$ are the electromagnetic and the weak parts of the quark neutral-current, respectively, shown in the following equations:

$$j^\mu_{\gamma,Q} = \sum_{\text{3 generations}} \frac{2}{3}\overline{u}\gamma^\mu u - \frac{1}{3}\overline{d}\gamma^\mu d, \tag{3.42}$$

$$j^\mu_{Z,Q} = \sum_{\text{3 generations}} 2g_L^U\overline{u_L}\gamma^\mu u_L + 2g_R^U\overline{u_R}\gamma^\mu u_R + 2g_L^D\overline{d_L}\gamma^\mu d_L + 2g_R^D\overline{d_R}\gamma^\mu d_R. \tag{3.43}$$

Feynman rules for quark neutral currents can be described as the leptonic ones. The ones that really change in the SM are the charged currents for quarks, and their vertex Feynman rules can be seen in figure (3.4).

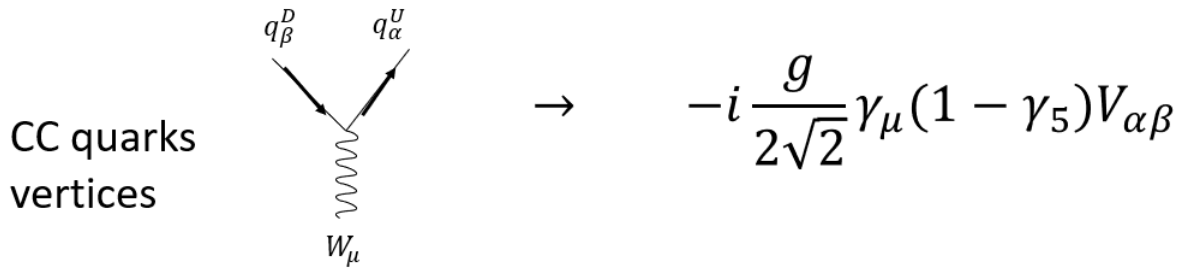

Figure 3.4: Vertex Feynman rules for quarks in charged currents.

3.2 Minimal gauge extensions to the Standard Model

The fermion mass hierarchy problem in the Standard Model relies mainly in that the three generations of fermions are identical in the charge assignation under the symmetry group. One way to avoid this is to extend the gauge symmetry into $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. This is called a minimal gauge extension, because the simplest extension is made with an abelian group $U(1)_X$. Distinction between generations is then possible, but new problems are brought to the discussion. Chiral anomalies are generally present, and for guaranteeing that the theory is renormalizable, they have to be identically cancelled. Naming X_f the charge under the appended symmetry group for a fermion field f, the following equations have to be fulfilled in order to have an anomaly free model:

$$
\begin{aligned}
&[\mathrm{SU}(3)_C]^2\,\mathrm{U}(1)_X \rightarrow A_C = \sum_Q (X_{Q_L} - X_{Q_R}) = 0 \\
&[\mathrm{SU}(2)_L]^2\,\mathrm{U}(1)_X \rightarrow A_L = \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} = 0 \\
&[\mathrm{U}(1)_Y]^2\,\mathrm{U}(1)_X \rightarrow A_{Y^2} = \sum_{\ell,Q} \left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L} - Y_{\ell_R}^2 X_{L_R} - 3Y_{Q_R}^2 X_{Q_R}\right] = 0 \\
&\mathrm{U}(1)_Y\,[\mathrm{U}(1)_X]^2 \rightarrow A_Y = \sum_{\ell,Q} \left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2 - Y_{\ell_R} X_{\ell_R}^2 - 3Y_{Q_R} X_{Q_R}^2\right] = 0 \\
&[\mathrm{U}(1)_X]^3 \rightarrow A_X = \sum_{\ell,Q} \left[X_{\ell_L}^3 + 3X_{Q_L}^3 - X_{\ell_R}^3 - 3X_{Q_R}^3\right] = 0 \\
&[\mathrm{Grav}]^2\,\mathrm{U}(1)_X \rightarrow A_\mathrm{G} = \sum_{\ell,Q} \left[X_{\ell_L} + 3X_{Q_L} - X_{\ell_R} - 3X_{Q_R}\right] = 0,
\end{aligned}
\tag{3.44}
$$

When the charges under $U(1)_X$ are different for particles of the same type but different generation, the symmetry $U(1)_X$ is said to be *non-universal*. Fermion mass hierarchy was perhaps firstly described phenomenologically by H. Fritzsch [71]. According to him, mass matrices that give naturally a fermion mass hierarchy should have a form like the one in the equation (3.45), called nowadays the *Fritzsch zero texture*. The purpose of charge assignation under $U(1)_X$ is that the symmetry itself forbids some Yukawa coupling, obtaining such a texture in the mass matrices.

$$
\begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix}
\tag{3.45}
$$

The $U(1)_X$ includes a Z' boson that has to get mass at a not reached energy scale, since it has not been observed. For this reason, minimal extensions include a scalar $SU(2)_L$ singlet field χ with a free parameter VEV v_χ. It should be a singlet, because otherwise it would contribute to the electroweak sector. Additional Higgs doublets can help in stablishing different scales of energy within the electroweak range, helping the fermion mass hierarchy of the known particles. Assume a model that has n scalar doublets similar to the Higgs doublet in SM and each one with with a VEV:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1+h_1}{\sqrt{2}} \end{pmatrix}, \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2+h_2}{\sqrt{2}} \end{pmatrix}, ..., \Phi_n = \begin{pmatrix} \phi_n^+ \\ \frac{v_n+h_n}{\sqrt{n}} \end{pmatrix}. \tag{3.46}$$

The scalar potential is very complex in such models, but some conditions have to be fulfilled for the sake of consistency with experimental observations so far:

- The VEV from the SM has to be recovered in order to give right masses to W and Z bosons. Therefore, $\sum_{i=1}^{n} v_i^2 = v^2$.
- Within the mass eigenstates, there should be one charged golstone boson for giving mass to the W and two pseudoscalar goldstone bosons for giving masses to the Z and the Z'.
- The lightest massive scalar particle is the candidate for being the Higgs boson. Therefore, its mass should depend directly on the SM's VEV v. Corrections with other parameters like v_χ are allowed but they should depend inversely on this in order to be suppressed.
- Other massive scalar eigenstates, which have not been observed, should lie naturally far beyond the electroweak energy scale. New parameters play a fundamental role here.

Acomplishing for FMH and for anomaly cancellation is usually not possible with the particles in the Standard Model, and therefore exotic fermion fields have to be considered. These new fields should be singlets under $SU(2)_L$, to ensure that they get masses at energy scales above the TeV. In that way, they do not couple with Higgs doublets but with the scalar singlet χ. The model of the thesis is a supersymmetric version of another model which is minimally gauge extended and whose aim was to achieve naturally FMH. The previous model was introduced by S. F. Mantilla and R. Martínez in 2017, and it satisfies a local gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ plus a parity symmetry $\mathbb{Z}_2$.Its fermion content can be seen in table (3.2), and it can be seen that both lepton and quark sectors are non-universal under $U(1)_X$. Moreover, different parity charges were assigned, because it helped in the zero textures of the fermion mass matrices. There is a whole new set of exotic particles, including down-like quarks, up like quarks ande charged leptons. Moreover, the previous model pursuited the explanation of neutrino masses through the iverse seesaw mechanism (ISS), and thereby, right-handed dirac neutrinos and Majorana neutrinos are regarded. The scalar content of the previous model is given in the table (3.3), and for making the different scalar fields to couple with specific generations of fermions, this sector is also non-universal under $U(1)_X$. Moreover, the scalar fields have also different parity charges. The scalar sector includes the usual scalar singlet χ for breaking the symmetry $U(1)_X$ and giving masses to exotic fermions. There are two scalar

Table 3.2: Fermion content of an non-universal $U(1)_X$ abelian extension to the Standard Model, with X quantum number and parity $\mathbb{Z}_2$ for each fermion.

Left-Handed Fermions	$X^{\pm}$	Right-Handed Fermions	$X^{\pm}$
SM Quarks			
$q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$	$+1/3^+$	u_R^1	$+2/3^+$
		u_R^2	$+2/3^-$
$q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$	0^-	u_R^3	$+2/3^+$
		d_R^1	$-1/3^-$
$q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$	0^+	d_R^2	$-1/3^-$
		d_R^3	$-1/3^-$
SM Leptons			
$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0^+	ν_R^e	$+1/3^-$
		ν_R^μ	$+1/3^-$
$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ \mu^\mu \end{pmatrix}_L$	0^+	ν_R^τ	$+1/3^-$
		e_R^e	$-4/3^-$
$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ \tau^\tau \end{pmatrix}_L$	-1^+	e_R^μ	$-1/3^-$
		e_R^τ	$-4/3^-$
Non-SM Quarks			
$\mathcal{T}_L$	$+1/3^-$	$\mathcal{T}_L^c$	$+2/3^-$
$\mathcal{J}_L^1$	0^+	$\mathcal{J}_R^1$	$-1/3^+$
$\mathcal{J}_L^2$	0^+	$\mathcal{J}_R^2$	$-1/3^+$
Non-SM Leptons			
E_L	-1^+	E_R	$-2/3^+$
$\mathcal{E}_L$	$-2/3^+$	$\mathcal{E}_R$	-1^+
Majorana Fermions		$\mathcal{N}_R^{1,2,3}$	0^-

Table 3.3: Scalar content of the model, non-universal X quantum number, $\mathbb{Z}_2$ parity and hypercharge

Higgs Scalar Doublets	$X^{\pm}$	Y	Higgs Scalar Singlets	$X^{\pm}$	Y
$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$	$+1$	$\chi = \frac{\xi_\chi+v_\chi+i\zeta_\chi}{\sqrt{2}}$	$-1/3^+$	0
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$+1/3^-$	$+1$	$\sigma = \frac{\sigma_\chi+i\zeta_\sigma}{\sqrt{2}}$	$-1/3^-$	0

doublets Φ_1 and Φ_2, each one with a different $U(1)_X$ charge. With them, only quarks c, t and b and charged leptons μ and τ aquire masses at the electroweak scale v. That is why there is an extra scalar singlet σ with no VEV, whose inclusion gives mass to the other particles through radiative corrections, giving masses to them proportional to v^2/v_x. In this way, the lightest fermion particles have their masses suppressed compared to the heavier ones.

Charged bosons W have a mass like in the SM in this model. Neutral bosons, on the other way, have an state adding to the mixing. First of all, the mass of these bosons are

$$M_\gamma = 0, \tag{3.47}$$

$$M_Z \approx \frac{gv}{2\cos\theta_W}, \tag{3.48}$$

$$M_{Z'} \approx \frac{g_X v_\chi}{3}, \tag{3.49}$$

where g_X is the $U(1)_X$ coupling constant. Mass eigenstates and gauge boson states are related by the following equation:

$$\begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W & 0 \\ \cos\theta_W\cos\theta_Z & -\sin\theta_W\cos\theta_Z & \sin\theta_Z \\ -\cos\theta_W\sin\theta_Z & \sin\theta_W\sin\theta_Z & \cos\theta_Z \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \\ B'_\mu \end{pmatrix}, \tag{3.50}$$

with $\sin\theta_Z$ a new mixing angle given by:

$$\sin\theta_Z = (1+\cos^2\beta)\frac{2g_X\cos\theta_W}{3g}\left(\frac{M_Z}{M_{Z'}}\right)^2. \tag{3.51}$$

The angle $\tan\beta = \frac{v_1}{v_2}$ has been also defined. Stating the general remarks and results from the model on which a supersymmetric extension was made, what is left is just to explain how this supersymmetrization is generally done.

Chapter 4

The Minimal Supersymmetric Standard Model

4.1 Motivations for supersymmetry

Although no signs of supersymmetry have been yet observed in experiments, a vast number of theoretical physicists continue defending it. The symmetry between fermions and bosons lead to some theoretical advantages. For this work the most relevant reason is the better agreement with the reported 125 GeV Higgs boson, compared to the Standard Model in terms of naturalness. However, more motivations are to be reviewed here since they could trace a path for possible future explorations of the present work. The first of them arose in the early years of supersymmetry, in a context of a ferverishly search for an internal symmetry that could contain or lead to the Poincaré symmetry.

4.1.1 The Colemann-Mandula and the Haag, Lopusański, Sohnius theorems

The Colemann-Mandula theorem [72] states that if a symmetry group of the S-matrix G fulfills the following conditions,

- Lorentz invariance: the Poincaré group is contained in G.
- Particle finiteness: The particle spectrum below any mass M is finite.
- Weak elastic analycity.
- Ocurrence of scattering.
- That G is representable under integral operators in the momentum space,

then G is locally isomorphic to a direct product of the Poincaré group and an internal symmetry group. In their work, they also remark a corollary impliying that all particles in a multiplet under the internal symmetry have the same mass. This last repercusion of the theorem can be seen by looking at the algebra of the operators involved [73]. Recalling, the algebra of the Poincaré group is:

$$
\begin{aligned}
&[P_\mu, P_\nu] = 0 \\
&[P_\mu, M_{\nu\lambda}] = i(\eta_{\mu\nu}P_\lambda - \eta_{\mu\lambda}P_\nu) \\
&[M_{\mu\nu}, M_{\lambda\rho}] = i(\eta_{\mu\rho}M_{\nu\lambda} - \eta_{\nu\rho}M_{\mu\lambda} - \eta_{\mu\lambda}M_{\nu\rho} + \eta_{\nu\lambda}M_{\mu\rho}),
\end{aligned} \tag{4.1}
$$

where P stands for the translation operator and M the space-time rotation operators. Let F_a be the operators of an internal symmetry. Its Lie algebra can be expressed as:

$$
[F_a, F_b] = iC^c_{ab}F_c, \tag{4.2}
$$

with C^c_{ab} the structure constant. Due to the theorem, the operators of the internal symmetry commute with the Poincaré operators. Defining the Casimir operators of the Poincaré group, which are invariant under group transformations, we identify the spin and the mass of the particles:

$$
P^2 = -m^2, \qquad\qquad U^2 = -m^2L, \tag{4.3}
$$

where $U^\mu = -\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}P_\mu M_{\lambda\rho}$, the mass is m and $\vec{L} = (M_{23}, M_{31}, M_{12})$ is the angular momentum. Since F_a commutes with the operators of the Poincaré group, they also commute with the Casimir operators defined above. Thus, all particles in a same representation of the internal symetry group have the same mass and the same spin.

Colemann-Mandula theorem states the imposibillity of merging Poincaré symetry and any internal symetry within a fundamental non-trivial representation. Its corollary shows that there is no way to find an internal symmetry that can change the spin or the mass of the particles. However, there is an assumption made in the theorem: The symmetries are Lie groups. A more general research was made by Haag, Lopusański and Sohnius [74], where they explore the validity of Colemann-Mandula theorem in what they called *pseudo-Lie algebras*, which are now known as graded Lie algebras or superalgebras. The generators explored in their paper for a massive theory are the ones from the Poncaré group in addition with spinor generators notated as Q^L_α and $\bar{Q}^L_{\dot{\alpha}}$, where $L = 1, ..., \nu$ and $\alpha = 1, 2$. For a massless theory, the spinor generators are rank 1, so $\alpha = 1, 2, 3, 4$. In the massless case, they show that a fusion between internal and geometrical symmetries is totally possible since the anticommutator of two spinor generators leads to a space-time translation. On the other hand, spinor generators are able to change spin within the same supermultiplet, but not the mass:

$$
[Q^L_\alpha, U^2] \neq 0, \qquad\qquad [Q^L_\alpha, P^2] = 0 \tag{4.4}
$$

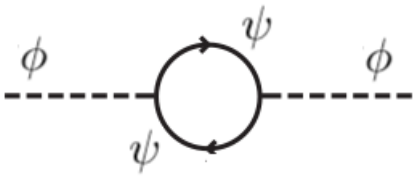

Figure 4.1: Fermion loop corrections to the squared mass of a scalar field.

4.1.2 The Higgs mass under an effective field theory: A naturalness problem

The observed 125 GeV Higgs boson is satisfactorilly explained in the SM, even when radiative corrections are regarded. However, it is generally accepted that the SM is far from being the fundamental theory of physics; it does not consider gravity within its framework and it lacks of a mechanism for explaining neutrino masses. Although no clear evidence of physics beyond the SM has been observed yet in colliders, an useful practice is to consider the SM as an effective field theory (EFT). In such a case, degrees of freedom of a high-energy physics are regarded to see how it affects the SM as a low energy limit.

However, there is a problem with scalar particles in EFTs. Scalar particles happen to get masses via radiative corrections that are sensitive to high-energy physics parameters, contradicting the very nature of an effective field theory. Thereby, there is no predictability. This is understood as a *naturalness* problem, since there are correlations within the theory between two widely sepparated energy scales. Porter Williams clarified in 2015 what it is understood by naturalness in this sense, because there is a general confussion in the literature with the very concept [75]. He showed as an example a model with just a fermion ψ of bare mass M and a scalar ϕ of bare mass m, interacting together similar as it happens in the SM after the SSB, where the Lagrangian was:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + i\bar{\psi}\not{\partial}\psi - M\bar{\psi}\psi + g\phi\bar{\psi}\psi \tag{4.5}$$

In an EFT there is a cut-off energy scale Λ that exclude field modes with energies higher than such a limit. If one assumes that the fermion field ψ stablish a high-energy physics scale but lower than the cut-off, $\Lambda > m \gg M$, a low energy limit can be done by integrating out the fermion field. The bare parameters g and m^2 get corrections from the fermion field at low energy. For the bare quadratic mass m^2, such corrections can be represented by a Feynmann diagram shown in the figure (4.1). Naming $(m^2)^*$ the corrected squared mass, it is given by:

$$(m^2)^* = m^2 + \frac{g}{16\pi^2}\left[\Lambda^2 + M^2 + m^2\ln\left(\frac{\Lambda}{M}\right) + \mathcal{O}\left(\frac{M^2}{\Lambda^2}\right)\right] \tag{4.6}$$

The corrected squared mass is not proportional to the bare mass and depends quadratically on M and the cut-off. However, the terms depending on Λ are result of a poor choose of the regularization scheme. When dimensional regularization is used, such terms dissappear. Although the problem seemed solved, there is still a dependece on M^2, remaining the sensitivity on high-energy physics. Supersymmetry has been mainly motivated for this reason. In fact, if a boson superparter $\tilde{\psi}$ is regarded in the Lagrangian (4.5), the term with M^2 is cancelled. Supersymmetric models are then suitable for studying EFT,

since there are no correlations between widely separated energy scales.

Supersymmetry requires fermion operators in order to be built. Thus, it falls beyond the Lie algebras to an extended framework, called the *graded Lie algebras*.

4.2 Graded Lie algebras

A graded algebra L is an algebra formed as the direct sum of $N+1$ subspaces L_k, with $N \geq 1$ [76].

$$L = \bigoplus_{k=0}^{N} L_k \tag{4.7}$$

If a product $\circ$ is defined in such an algebra, it fulfills that: for $u_i \in L_i$ and $u_j \in L_j$, the product is a *grading*,

$$u_i \circ u_k \in L_{i+jMod(N+1)}. \tag{4.8}$$

In this work only $N = 1$ algebras are going to be considered. This graded algebras are usually called $\mathbb{Z}_2$-*graded algebras*. For a graded algebra to be a graded Lie algebra, two more properties are followed by a grading. The first of them is known as the supersymmetrization: $\forall x_i \in L_i$ and $\forall x_j \in L_j$, $i, j = 0, 1$:

$$x_i \circ x_j = (-1)^{ij} x_j \circ x_i, \tag{4.9}$$

where for $i, j = 1$ the product can be identified with an anticommutator and for the rest of the cases, with a commutator. This is why the property is called supersymmetrization, since commutators describe bosons and anticommutators describe fermions. The second property is that the product satisfies a generalized Jacobi identity. $\forall x_i \in L_i$, $\forall x_j \in L_j$ and $\forall x_k \in L_k$, $i, j, k = 0, 1$, the identity reads:

$$x_k \circ (x_i \circ x_j)(-1)^{kj} + x_i \circ (x_j \circ x_k)(-1)^{ik} + x_j \circ (x_k \circ x_i)(-1)^{ij} = 0. \tag{4.10}$$

This is necessary to contain a Lie algebra. In fact, for a $\mathbb{Z}_2$-graded Lie algebra, L_0 is a Lie algebra itself and L_1 is not an algebra at all, because it is not closed under products. Operators from L_1 can be spinor operators, but first the general framework of spinors has to be detailed.

4.3 Spinor algebra

Spinors are representations of the special linear two-dimensional complex group $SL(2, \mathbb{C})$. With special, it is referring to elements with determinant $+1$. This group admits two

inequivalent linear representations: [76]

- The self-representation: Let $M \in SL(2,\mathbb{C})$. The self representation is just $D(M) = M$.
- The complex conjugate self-representation: Let $M \in SL(2,\mathbb{C})$. The complex conjugate self representation is $D(M) = M^*$.

Since the group is two dimensional, the inequivalent representations just listed above are two dimensional as well. For the self-representation and the complex conjugate self-representation let us denote the corresponding vector spaces as F and $\dot{F}$, respectively. Spinors $\psi \in F$ are called *left-handed Weyl spinors* and they transform under the group as:

$$\psi'_A = M_A{}^B \psi_B \quad A, B = 1, 2. \tag{4.11}$$

The elements of the complex conjugate self-representation are called *right-handed Weyl spinors* and they transform under the group as:

$$\bar{\psi}'_{\dot{A}} = (M^*)_{\dot{A}}^{\dot{B}} \bar{\psi}_{\dot{B}} \quad \dot{A}, \dot{B} = 1, 2. \tag{4.12}$$

Dotted and undotted indices are used to distinguish between the two representations. There is an analogy here with covariant and contravariant indices from special relativity, and the reason of this is that there can be found equivalent representations that transform with inverse elements. For instance, the representation $D(M) = M^{-1T}$ is equivalent to the self-representation. They both can be related with an element of the group $\epsilon \in SL(2,\mathbb{C})$ as $\epsilon M \epsilon^{-1} = M^{-1T}$. It can be found that:

$$\epsilon^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{4.13}$$

With this, it is possible to define contravariant left-handed Weyl spinors, which are elements of the dual space F^*:

$$\psi^A = \epsilon^{AB} \psi_B. \tag{4.14}$$

The representation $D(M) = M^{*-1T}$ is equivalent to the complex conjugate self-representation and they are related by:

$$\epsilon^{\dot{A}\dot{B}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{4.15}$$

Similarly, one can define contravariant right-handed Weyl spinors, which are elements of

the dual space $\dot{F}^*$

$$\bar{\psi}^{\dot{A}} = \bar{\epsilon}^{\dot{A}\dot{B}}\bar{\psi}_{\dot{B}}. \tag{4.16}$$

From the perspective of linear algebra, the elements of the dual spaces F^* and $\dot{F}^*$ are linear maps from F and F^* to a field $\mathbb{C}$, respectively. Taking $\psi^A \in F^*$ and $\bar{\psi}^{\dot{A}} \in \dot{F}^*$, it can be seen how these maps act like:

$$\psi(\phi) = \psi_A\phi^A \in \mathbb{C}, \qquad \bar{\psi}(\bar{\phi}) = \bar{\psi}_{\dot{A}}\bar{\phi}^{\dot{A}} \in \mathbb{C}. \tag{4.17}$$

These mappings are analogous to the dot product, considering ϵ as the metric tensor. Since F and $\dot{F}$ are vector spaces from inequivalent representations, no mapping with an element of $SL(2,\mathbb{C})$ is possible. However, there can be a transition $F \to \dot{F}^*$ with the following operation:

$$(\bar{\sigma}^0)^{\dot{A}A}(\psi_A)^* = \bar{\psi}^{\dot{A}}, \tag{4.18}$$

with $(\bar{\sigma}^0)^{\dot{A}A} = \mathbb{I}_{2\times 2}$. Thereby, the four complex components $\psi_A, \bar{\psi}^{\dot{A}}$ correspond really to four real independent numbers.

Remarks about calculations with spinors

Each component of a spinor is to be thought as a Grassmann variable. If one considers each component as a complex number, then some invariant quantities under $SL(2\mathbb{C})$ would be identally zero. To see this, such invariant quantities are constructed in the following way: Then the invariant quantities are given right ahead,

$$(\psi\chi) := \psi^A\chi_A = \quad \text{and} \quad (\bar{\psi}\bar{\chi}) := \bar{\psi}^{\dot{A}}\bar{\chi}_{\dot{A}}. \tag{4.19}$$

Spinor indices were omitted in the contraction, and this notation will be used from now on. Explicitly and for instance, $(\psi\psi) = \psi_A\psi^A = \psi_2\psi_1 - \psi_1\psi_2$. If the numbers were regarded as complex conmuting numbers rather than Grassmann numbers, the invariant term would be identically zero. In the following lines, there are some properties that are used for calculations with spinors. Let ψ, χ and ϕ be left-handed spinors, then:

- The quadratic forms are symmetrical in exchange: $(\psi\chi) = (\chi\psi)$ and $(\bar{\psi}\bar{\chi}) = (\bar{\chi}\bar{\psi})$.
- Other quadratic forms can be related to the invariant forms: $\psi_A\psi_B = \frac{1}{2}\epsilon_{AB}(\psi\psi)$ and $\bar{\psi}^{\dot{A}}\bar{\psi}^{\dot{B}} = \frac{1}{2}\epsilon^{\dot{A}\dot{B}}(\psi\psi)$.
- The Fierz reordering formula for Weyl spinors is: $(\psi\chi)(\psi\phi) = -\frac{1}{2}(\psi\psi)(\chi\phi)$.
- The generators of the group $SL(2,\mathbb{C})$ in the spinor representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ are given by $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\mu\bar{\sigma}^\nu)$ and $\bar{\sigma}^{\mu\nu} = \frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\nu)$, respetively.

- The rotation $\tilde{J}_i$ and Lorentz boost $\tilde{J}_i$ can be written as $\tilde{J}_i = \frac{1}{2}\sigma^i\bar{\sigma}^0$ and $\tilde{K}_i = -\frac{i}{2}\sigma^i\bar{\sigma}^0$.
- The term $\phi\sigma^\mu\bar{\chi}$ is a Lorentz vector, which means that under Lorentz transformations $\phi\sigma^\mu\bar{\chi} \to \lambda(M)\phi\sigma^\mu\bar{\chi}$.

Spinors are conformed by Grassmann numbers. That makes them paramount if one defines a field theory in a way that supersymmetry comes from symmetries of an artifitial space-time with spinor coordinates. Before that, Grassmann variables must be dicussed.

4.4 Grassmann variables

4.4.1 Grassmann algebras

A Grassmann algebra has a finite set of n elements $\{\eta_1, ..., \eta_n\}$ that fulfill two conditions: [77]

$$\eta_i^2 = 0 \qquad \text{and} \qquad \{\eta_i, \eta_j\} = \eta_i\eta_j + \eta_j\eta_i = 0 \quad \text{for i,j=1, ..., } n. \tag{4.20}$$

It is possible to regard the set of elements as a vector space and define a vector a as a linear combination,

$$a = a_1\eta_1 + ... + a_n\eta_n, \tag{4.21}$$

where the coeficients a_i can be considered as complex or real. Defining in a similar way a vector b, it can be shown that the vectors inherit the properties of the Grassmann algebra:

$$a^2, b^2 = 0 \qquad \text{and} \qquad \{a, b\} = 0. \tag{4.22}$$

Explicitly and taking into account the anticommutativity of the elements of the set, the product amongst two vectors turns out to be:

$$ab = \sum_{i,j}^{n} \frac{1}{2}(a_ib_j - a_jb_i)\eta_i\eta_j \tag{4.23}$$

The algebra is therefore more complex than the single set, since other elements can be obtained through products of the elements of the set. In this sense, the Grassmann algebra can be regarded as graded algebra. The multiplication of the r elements is said to be an element of grade r.

4.4.2 Grassmann differentiation

It is possible to define both integral and differential calculus on a set of Grassmann numbers $\{\eta_1, \eta_2, ..., \eta_N\}$. For the derivative, the first formal definition states the idependence amongst the N variables,

$$\frac{\partial a_i}{\partial a_j} := \delta_{ij}. \tag{4.24}$$

Derivatives are not to be understood here as ratios of infinitesimals. The next is to stablish a product rule that does not spoil the anticommuting properties,

$$\frac{\partial}{\partial a_p}(a_{i1}, a_{i2}, ..., a_{ir}) = \delta_{pi1} a_{i2}...a_{ir} + ... + (-1)^{r-1}\delta_{pir} a_{i1}...a_{ir-1}. \tag{4.25}$$

Let $f(\eta_1, .., \eta_N)$ be a function of the Grassmann numbers. It can be shown that these types of derivatives just defined anticommute,

$$\left\{ \frac{\partial}{\partial \eta_i}, \frac{\partial}{\partial \eta_j} \right\} f(\eta_1, .., \eta_N) = 0. \tag{4.26}$$

Thus, derivative operators for Grassmann numbers behave aswell as such numbers. A differential calculus for spinors can be extended from here. Considering the Weyl spinors θ^A and $\bar{\theta}^{\dot{A}}$, we define the derivative operators as:

$$\partial_A := \frac{\partial}{\partial \theta^A}, \qquad \bar{\partial}_{\dot{A}} := \frac{\partial}{\partial \bar{\theta}^A}, \tag{4.27}$$

where one can raise the index by using the ϵ matrix. Taking account that the spinor components are Grassmann numbers, the following properties are easily derived:

$$\partial_A \theta^B = \delta_A^B, \qquad \bar{\partial}_{\dot{A}} \bar{\theta}^{\dot{B}} = \delta_{\dot{A}}^{\dot{B}}. \tag{4.28}$$

In the same way as Grassmann derivatives, the spinor derivatives also show an anticommutative behavior:

$$\{\partial_A, \partial_B\} = 0, \qquad \{\bar{\partial}_{\dot{A}}, \bar{\partial}_{\dot{B}}\} = 0. \tag{4.29}$$

4.4.3 Grassmann integration

The Grassmannn integration, which happens to be a Berenzin integral, is usually denoted with the following symbol:

$$\int daf(a) = I[f] \tag{4.30}$$

Since the variables in which it depends are not real numbers, the integral shall not be interpreted as an area below a curve nor has sense to define limits. The following properties are demanded:

- $\int daf(a+b) = \int daf(a)$.
- Linearity: let $\alpha, \beta \in \mathbb{C}$, then $\int da[\alpha f(a) + \beta g(a)] = \alpha \int daf(a) + \beta \int dag(a)$.

Function can be integrated if a Taylor-like expansion is made, considering aswell that $a^2 = 0$.

$$f(a) = f(0) + f^{(1)}(0)a \tag{4.31}$$

Defining $I[f] = f^{(1)}(0)$, then:

$$I[f] = f(0)\int da\mathbb{I} + f^{(1)}\int daa := f^{(1)} \longrightarrow \int da\mathbb{I} = 0 \quad \text{and} \quad \int daa = 1. \tag{4.32}$$

Considering a more complex Grassmann algebra G_2 generated by the elements θ_1 and θ_2, there are four independent elements $\mathbb{I}_{G_2}, \theta_1, \theta_2, \theta_1\theta_2$. Anticommutation behavior amongst the differentials is imposed:

$$\{d\theta_1, d\theta_2\} = \{d\theta_1, \theta_2\} = 0. \tag{4.33}$$

To find the integrals in this algebra, consistency with the previous definition is demanded, impliying some other results:

$$\int d\theta_1 \int d\theta_2\theta_1 = \int d\theta_1 \int d\theta_2 = 0 \quad \text{and} \quad \int d\theta_1 \int d\theta_2\theta_1\theta_2 = -1 \tag{4.34}$$

Now it is possible to interpret the integral of a function defined in this algebra $f(\theta_1, \theta_2)$ as a projection in the coefficient of the highest term of its Taylor-like expansion,

$$\int d\theta_1 \int d\theta_2 f(\theta_1\theta_2) = \int d\theta_1 \int d\theta_2[f^{(0)} + \theta_1 f^{(1)} + \theta_2 f^{(2)} + \theta_2\theta_3 f^{(3)}] = -f^{(3)} \tag{4.35}$$

Delta-like functions can also be found and defined as proper integration deltas. For

algebras like G it can be seen that $\delta(a) := a$ fulfills:

$$\int da f(a)\delta(a) = f(0) \tag{4.36}$$

A supersymmetric field theory can be seen as a field theory over a mathematical extension of the Minkowski space-time, where Grassmann dimensions expressed as spinor elements are appended; this is called the *superspace*. An action functional of this type of fields can be considered in the superspace, exhibited in the integration over the spinor coordinates. The following lines of this subsection will describe general features of integration over such an space. First of all, differential elements of volume of the superspace are given by:

$$\begin{aligned} &d^2\theta := -\frac{1}{4} d\theta^A d\theta^B \epsilon_{AB}, \qquad d^2\bar{\theta} := -\frac{1}{4} d\bar{\theta}_{\dot{A}} d\bar{\theta}_{\dot{B}} \epsilon^{\dot{A}\dot{B}}, \\ &d^4\theta := d^2\theta d^2\bar{\theta}. \end{aligned} \tag{4.37}$$

This definitions are just to give consistency with the other Grassmann integrations. This can be seen in the following implications:

$$\int d^2\theta(\theta\theta) = 1, \quad \int d^2\bar{\theta}(\bar{\theta}\bar{\theta}) = 1, \quad \int d^2\theta = \int d^2\bar{\theta} = \int d^2\theta\theta_A = \int d^2\bar{\theta}\bar{\theta}_{\dot{A}} = 0. \tag{4.38}$$

Finally, the deltas of integration happen to be:

$$\delta^2(\theta) = (\theta\theta) \qquad\qquad \delta^2(\bar{\theta}) = (\bar{\theta}\bar{\theta}) \tag{4.39}$$

4.5 Superfield theory

Before introducing the formalism of superfields, a supersymmetric toy model is going to be presented in two different ways.

4.5.1 On shell and off shell supersymmetry

A supersymmetric field theory has for every fermionic degree of freedom a bosonic one, and viceversa. This means that the action describing the fields is invariant under transformations that relate fermions and bosons. To understand what supersymmetry implies, the two-dimensional Wess-Zumino model can be used as a toy model. The action is only composed by the kinematic terms of a scalar boson field ϕ and a fermion field ψ,

$$S_{WZ} = -\frac{1}{2}\int dx^2[\bar{\psi}\partial\!\!\!/\psi + \partial_\mu\phi\partial^\mu\phi], \tag{4.40}$$

Supersymmetry holds if the action, or at least the equations of motion, are invariant under transformations that take boson fields to fermion fields and viceversa. Infinitesimal supersymmetric transformations on the fields have the following form:

$$\begin{aligned} \delta\phi &= \bar{\epsilon}\psi \\ \delta\psi &= \partial\phi\epsilon, \end{aligned} \tag{4.41}$$

where ϵ is a spinor parameter of the transformations. The number of spinor parameters can be many, but in this case there is only one. This is called $N = 1$ supersymmetry. The action (4.40) turns out to be invariant under the supersymmetric transformations (4.41). The supersymmetry here is present if the equations of motion of the fields, $\partial\psi = 0$ and $\Box\phi = 0$, are imposed. This is because in this way the fields have the same number of degrees of freedom. This is called *on-shell* supersymmetry.

By adding to the action (4.40) a scalar field F with the term $\int dx^2 F^2$, one can extend the supersymmetry without affecting the equations of motion, since the equation of motion for the new field is $F = 0$. This is called *off-shell* supersymmetry [78], and it allows the extrapolation to the superfield formalism, which is very useful in $N = 1$ supersymmetry. The superfield formalism involves the fermion, boson and auxiliary fields like F within one mathematical object.

4.5.2 Supersymmetry as a symmetry of the superspace

Working with a supersymmetric action, like in the case of off-shell supersymmetry, can give a compact way of describing a supersymmetric model through the superfield formalism. In such a description, one can consider fields defined in an extension of the Miskoswki space-time, called the superspace. The superspace then is conformed of the commuting coordinates x^μ and anticommuting grassmann coordinates. Grassmann coordinates are arranged as two-component Weyl-spinors, one left-handed and the other one right-handed:

$$\{\theta_A\}_{A=1,2} \quad \text{and} \quad \left\{\bar{\theta}_{\dot{B}}\right\}_{\dot{B}=1,2}, \text{ respectively.} \tag{4.42}$$

Changes of the spinor coordinates are carried by operators Q_A and $\bar{Q}_{\dot{B}}$. In a similar way that the transformations of the coordinates x^μ imply operators that satisfy the Poincaré algebra, the operators Q_A and $\bar{Q}_{\dot{B}}$ satisfy the following algebra:

$$\begin{aligned} \{Q_A, Q_B\} &= 0 \\ \left\{\bar{Q}_{\dot{A}}, \bar{Q}_{\dot{B}}\right\} &= 0 \\ \left\{Q_A, \bar{Q}_{\dot{B}}\right\} &= 2\sigma^\mu_{A\dot{B}} P_\mu \end{aligned} \tag{4.43}$$

4.5.3 Superfields

A superfield is a quantity that contains both a fermion, a boson and an auxiliary field, that is defined in the superspace. Such quantities will be denoted by a hat symbol, for distinguishing them from ordinary fields. Let us now determine the different kinds of field representations that are used in a $N = 1$ supersymmetry. For defining the quantities that contain scalar and fermion fields, we must show the covariant derivatives that are invariant under supersymmetric transformations:

$$D_A = \frac{\partial}{\partial\theta^A} + i\sigma^\mu_{A\dot{A}}\bar{\theta}^{\dot{A}}\frac{\partial}{\partial x^\mu} \qquad \bar{D}_{\dot{A}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{A}}} - i\theta^A\sigma^\mu_{A\dot{A}}\frac{\partial}{\partial x^\mu}, \tag{4.44}$$

where σ^μ are the three Pauli matrices and the $\sigma^0 = \mathbb{I}_{2\times2}$. These derivatives define projection operators, which separate superfields into two subsets: Left-handed chiral superfield and right-handed chiral superfield, which are defined respectively as:

$$\bar{D}_{\dot{A}}\hat{\Phi}_L(x,\theta,\bar{\theta}) = 0 \rightarrow \hat{\Phi}_L(x,\theta,\bar{\theta}) = A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \tag{4.45}$$

$$D_A\hat{\Phi}_R(x,\theta,\bar{\theta}) = 0 \rightarrow \hat{\Phi}_R(x,\theta,\bar{\theta}) = A^*(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) + \bar{\theta}\bar{\theta}F(x) \tag{4.46}$$

The chiral superfields were Taylor expanded in the Grassmann variables with the conditions given above. It can be seen that each one contains a scalar field $A(x)$, a spinor field $\psi(x)$ and an auxiliary field $F(x)$. On the other hand, the gauge fields can be obtained through the vector supefields, defined only by the reality condition $V^\dagger(x,\theta,\bar{\theta}) = V(x,\theta,\bar{\theta})$. After imposing the *Wess-Zumino gauge*, which retains the essential fields of the superfield expansion for a supersymmetric gauge transformation, the Taylor expansion reads:

$$\hat{V}(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}V_\mu(x) + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\lambda(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x) \text{ Vector superfield} \tag{4.47}$$

Thereby, $\hat{V}(x,\theta,\bar{\theta})$ contains a vector field $V_\mu(x)$, a spinor field $\lambda(x)$ and a scalar auxiliary field $D(x)$. The scalar auxiliary fields $D(x)$ and $F(x)$ have no degrees of freedom in the on-shell picture, but they are important for defining the action of a supersymmetric theory [76]. The Wess-Zumino gauge is going to be assumed for vector superfields in the rest of the theoretical frame. A supersymmetric action has an integral over the Grassmann variables as well, but this one can be easily made, so one ends up with an action on the Minkowski coordinates only. Therefore, it is important to express the following superfield products that survive to the Grassmann integration:

$$\begin{aligned}
&\hat{\Phi}_i\hat{\Phi}_j|_{\theta\theta\bar{\theta}\bar{\theta}} = A_iF_j + F_iA_j - \psi_i\psi_j, \\
&\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k|_{\theta\theta\bar{\theta}\bar{\theta}} = A_iA_jF_k + A_jF_jA_k + F_iA_jA_k - \psi_i\psi_kA_j - \psi_i\psi_jA_k - A_i\psi_j\psi_k.
\end{aligned} \tag{4.48}$$

The minimal coupling with gauge fields is introduced in the following form of the *Kähler potential*,

$$\mathcal{L}_{kin} = \sum_i \hat{\Phi}^i_R e^{q_i\hat{V}}\hat{\Phi}^i_L, \tag{4.49}$$

where $\hat{\phi}^i$ is a chiral superfield with its q_i charge, which corresponds to an interaction whose gauge field is contained in the $\hat{V}$ vector superfield. The index i stands in this subsection for any chiral superfield in the theory. While expanding using the products (4.48) and integrating the action in the Grassmann variables, kinetic terms of fermions

and their superpartners are obtained. An addiional term $\sum_i F_i F_i^*$ also remains, involving the auxiliary fields. The kinetic terms for the gauge fields come from a supersymmetric extension of the field strenght $W_A = -\frac{1}{4}\bar{D}\bar{D}D_A\hat{V}$. The supersymmetry invariant Lagrangian for this object is $W_A W^A \delta^2(\bar{\theta}) + \bar{W}_{\dot{A}}\bar{W}^{\dot{A}}\delta^2(\theta)$, and when expanding one obtains the kinetic terms of the gauge bosons, gauginos (the superpartners) and a term involving the auxiliary fields $\sum_j D_j D_j^*$. The index j stands in this subsection for every gauge field in the theory. The rest is to take up to trilinear terms for the renormalizability of the theory that respect the gauge and Poincaré symmetries. This is called the superpotential. The equations of motion for auxiliary fields are required for eliminating such degrees of freedom, and they are:

$$F_i(x) + W_i^*(x) = 0 \tag{4.50}$$
$$D_j^a = -g_j T_{lm}^a A_l^*(x) A_m(x), \tag{4.51}$$

where T_{lm}^a are the generators of the symmetry group corresponding to the gauge field and g_j is the coupling constant. It was also introduced $W_i = \frac{\partial W[A_1,A_2,...,A_n]}{\partial A_i}$, where $W[A_1, A_2, ..., A_n]$ is the superpotential. The superpotential is an holomorphic function (or analitically complex) of the scalar fields regarded as complex variables, from which non-gauge supersymmetric interactions are obtained. Usually one requires to break explicitly the supersymmetry since it has not been yet discovered. This is done by adding soft supersymmetry breaking mass terms, which raise for instance the mass of all superpartners.

4.6 Minimal Supersymmetric Standard Model

The most simple possible supersymmetric extension to the Standard Model is known as the Minimal Supersymmetric Standard Model (MSSM). It is constructed by promoting all SM fields to superfields, but including some important considerations that will be mentioned in its due moment. From now on superfields will be denoted by a hat symbol ˆ. First of all, terms in a superpotential like $\hat{H}\hat{H}^*$ or $\bar{\hat{u}}\hat{H}^*\hat{Q}$ are not holomorphic. Thereby, they are not allowed by supersymmetry. For giving masses to up quarks (and to neutrinos in a model with right-handed neutrinos), the MSSM works with two Higgs doublet model. The two Higgs supermultiplets are given right ahead:

$$\hat{H}_d = \begin{pmatrix} \hat{H}_d^+ \\ \frac{\hat{h}_d + v_d + \hat{\eta}_d}{\sqrt{2}} \end{pmatrix} \qquad \hat{H}_u = \begin{pmatrix} \frac{\hat{h}_u + v_u + \hat{\eta}_u}{\sqrt{2}} \\ \hat{H}_u^- \end{pmatrix}. \tag{4.52}$$

The superdoublet $\hat{H}_d$ has the same charge assignation under $SU(3)_C \times SU(2)_L \times U(1)_Y$ than the Higgs doublet in the SM and $\hat{H}_u$ has the opposite charges. This is also for cancelling chiral anomalies that come from the fermion superpartners of the scalar fields. The other Higgs doublet superfield enables the construction of the Higgs potential. The most general superpotential of the MSSM for one generation (other generations have similar terms) is given by [79]:

$$W_{MSSM} = \hat{\bar{u}} y_u \hat{Q}\hat{H}_u - \hat{\bar{d}} y_d \hat{Q}\hat{H}_d - \hat{\bar{e}} y_e \hat{L}\hat{H}_d + \mu \hat{H}_u \hat{H}_d + \text{B-L violating terms}, \tag{4.53}$$

where $\hat{L}$ and $\hat{Q}$ are chiral left-handed doublet superfields corresponding to leptons and quarks, respectively. Right-handed chiral superfields are given by $\hat{\bar{u}}$, $\hat{\bar{d}}$ and $\hat{\bar{e}}$. The first three terms are responsible of the fermion masses, since they are Yukawa couplings. The last term generates the Higgs potential. There are other possible terms that were not explicitly put in equation (4.53) but just refered as *B-L violating terms*. As its name indicates, they are plausible terms that respect supersymmetry but violate the baryion and lepton numbers, which is strongly forbidden. The superpotential related to this is given into two parts:

$$W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk}\hat{L}_i\hat{L}_j\hat{\bar{e}}_k + \lambda'^{ijk}\hat{L}_i\hat{Q}_j\hat{\bar{d}}_k + \mu'_i\hat{L}_iH_u \tag{4.54}$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{ijk}\hat{\bar{u}}_i\hat{\bar{d}}_j\hat{\bar{d}}_k. \tag{4.55}$$

The first equation accounts for term that lead to a lepton number violation of 1. The second one implies a baryon number violation of 1. This flaw is covered in the MSSM by introducing by hand a parity symmetry called the R-parity, which forbids terms like in equation (4.54). Another important defect of all supersymmetric models is that supersymmetry has not been yet observed. Therefore, every model, and in particular the MSSM, should include a Lagrangian with mass terms that respect the gauge symmetry but not the supersymmetry. This one is called soft-supersymmetry breaking potential and in the MSSM is given in the following lines,

$$\begin{aligned}\mathcal{L}_{MSSM-Soft} = &-\frac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + h.c)\\ &-(\tilde{\bar{u}}a_u\tilde{Q}H_u - \tilde{\bar{d}}a_d\tilde{Q}H_d - \tilde{\bar{e}}a_e\tilde{Q}H_d)\\ &-\tilde{Q}^\dagger m_Q^2\tilde{Q} - \tilde{L}^\dagger m_L^2\tilde{L} - \tilde{\bar{u}}^\dagger m_{\bar{u}}^2\tilde{\bar{u}} - \tilde{\bar{d}}^\dagger m_{\bar{d}}^2\tilde{\bar{d}} - \tilde{\bar{e}}^\dagger m_{\bar{e}}^2\tilde{\bar{e}}\\ &-m_{H_u}H_u^*H_u - m_{H_d}H_d^*H_d - (bH_uH_d + c.c)\end{aligned} \tag{4.56}$$

The first line corresponds to mass terms from supersymmetric partners of gauge bosons: gauginos. The second line consists of a Yukawa interaction for superpartners of the fermions. The third line are mass terms for superpartners of the fermions. Finally, the fourth line is made of supersymmetry breaking corrections to the Higgs potential. While the SM has 19 free parameters, the MSSM has 124 free parameters [80].

4.6.1 Scalar spectrum of the MSSM

From the superpotential and the soft-supersymmetry breaking terms, shown respectively in equations (4.53) and (4.56), the scalar potential of the MSSM reads [81]:

$$\begin{aligned}V_H = &(\mu^2 + m_{H_u}^2)|H_u|^2 + \frac{1}{8}(g^2+g'^2)(|H_u|^2)^2 + (\mu^2 + m_{H_d}^2)|H_d|^2 + \frac{1}{8}(g^2+g'^2)(|H_d|^2)^2\\ &-\frac{1}{4}(g^2+g'^2)|H_u|^2|H_d|^2 + \frac{g^2}{2}|\bar{H}_uH_d|^2 - (b\epsilon_{ij}H_u^iH_d^j + c.c),\end{aligned} \tag{4.57}$$

where $i, j = 1, 2$. It can be obtained by considering D-terms and F-terms, as it was explained in the previous section. In the doublets H_u and H_d there are the charged fields $H_u^{\pm}$ and $H_u^{\pm}$, the CP-even fields h_u and h_d, and the CP-odd fields η_u and η_d. Thus, from the potential (4.57), mass terms can be extracted and put in a matrix form. For the CP-odd field, the mass matrix from which the eigenvalues are the masses of pseudoscalar particles is shown below,

$$\mathcal{M}_\eta = \frac{b}{v_u v_d} \begin{pmatrix} v_u^2 & v_u v_d \\ v_u v_d & v_d^2 \end{pmatrix}. \tag{4.58}$$

Given that $det[\mathcal{M}_\eta] = 0$, there is a massless state that corresponds to a gauge boson, which gives mass to the Z boson. Moreover, the other eigenvalue can be calculated directly as the trace of the mass matrix. Naming A^0 as the massive eigenstate, its mass turns out to be:

$$m_{A^0}^2 = \frac{b}{v_u v_d}(v_u^2 + v_d^2) = \frac{2b}{\sin 2\beta}, \tag{4.59}$$

where β is an angle parameter defined by $\tan\beta \equiv v_d/v_u$. This mass depends on the mixing parameter b from the soft SUSY breaking potential. Thus, it is naturally much heavier than SM particles, escaping from experimental constraints. In the case of the charged fields, a mass matrix can also be found. When diagonalized, the mass eigenstates $H^{\pm}$ and $G^{\pm}$ can be related to $H_d^{\pm}$ and $H_u^{\pm}$ through the following equation:

$$\begin{pmatrix} H^{\pm} \\ G^{\pm} \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_d^{\pm} \\ H_u^{\pm} \end{pmatrix}, \tag{4.60}$$

where their masses are given by the next lines,

$$\begin{aligned} m_{G^{\pm}}^2 &= 0, \\ m_{H^{\pm}}^2 &= m_W^2 + m_{A^0}^2. \end{aligned} \tag{4.61}$$

As it happened for the CP-odd sector, there is a goldstone boson $G^{\pm}$ that gives mass to the $W^{\pm}$ and a heavy state $H^{\pm}$. Finally, the CP-even masses can be obtained from the mass matrix given ahead,

$$M_h^2 = \begin{pmatrix} m_{A^0}^2 \sin^2\beta + m_Z^2 \cos\beta & -(m_{A^0}^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_{A^0}^2 + m_Z^2)\sin\beta\cos\beta & m_{A^0}^2\cos^2\beta + m_Z^2\sin\beta \end{pmatrix}. \tag{4.62}$$

Thereby, the quadratic mass eigenvalues are:

$$m_{h_0}^2 = \frac{1}{2}\left[m_{A^0}^2 + M_Z^2 - \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2\cos^2(2\beta)}\right], \tag{4.63}$$

$$m_{H_0}^2 = \frac{1}{2}\left[m_{A^0}^2 + m_Z^2 + \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2 m_Z^2\cos^2(2\beta)}\right]. \tag{4.64}$$

Doing a Taylor expansion on the ratio m_Z/m_A until the fourth order, we have the following approximated forms:

$$m_{h_0}^2 \approx \cos^2(2\beta) m_Z^2 \left[1 - \frac{m_Z^2}{2m_{A^0}^2}(1 - \cos(2\beta))\right], \tag{4.65}$$

$$m_{H_0}^2 \approx m_{A^0}^2 + m_Z^2 - \cos^2(2\beta) m_Z^2 \left(1 - \frac{m_Z^2}{m_{A^0}^2} - \frac{m_Z^4}{m_{A^0}^4}\cos^2(2\beta)\right). \tag{4.66}$$

The lightest state is naturally at the electroweak scale at tree level and the other one escapes from experimental constraints, because its mass is proportional to m_{A^0}.

Upper limit of the MSSM lighests CP-even particle

At tree level the mass of the Higgs fulfills the condition:

$$m_{h_0} < \cos(2\beta) m_Z. \tag{4.67}$$

Since $m_Z \approx 91.2$ GeV, the Higgs mass turns out to fall short in the MSSM at tree level. Considering radiative corrections at one loop with the top quark and its superpartner, the Higgs mass is corrected:

$$m_h^2 = m_Z^2 \cos^2(2\beta) + \Delta m_h^2, \tag{4.68}$$

where Δm_h^2 has been found in literature to be the following expression [82]:

$$\Delta m_h^2 = \frac{3g^2 m_Z^4}{16\pi^2 m_W^2}\left[ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)\left(\frac{2m_t^4 - m_t^2 m_Z^2}{m_Z^4} + \frac{m_t^2}{m_Z^2}\right)\right], \tag{4.69}$$

where t and $\bar{t}$ correspond to the top quark and its superpartner, respectively. With radiative corrections, the mass of the Higgs can be risen up to 130 GeV [83]. This means that radiative corrections have to be larger than the contribution of the leading order. This is considered as unnatural.

Chapter 5

A $U(1)_X$ non-universal extension of the MSSM

The general aspects of supersymmetrization have been discussed in the previous chapter. An analogous prescript to the one followed for the MSSM was used to supersymmetrize the $U(1)_X$ non-universal model, described in section (3.2). This model has been already published by myself, Roberto Martínez and Sebastián Alvarado [1]. Thus, many of the results that are shown in that paper are going to be also seen here.

5.1 General overview

The supersymmetric model is constructed by firstly upgrading all fields to superfields. To avoid the appearance of chiral anomalies due to the new fermion counterparts of scalar fields, the Higgs sector was extended. Thus, for every field in this sector, there is another one with contrary charges under the symmetry group $SU(2)_L \times U(1)_Y \times U(1)_X$. The Higgs content of the model can be seen in the table (5.1). The superfields coming from fermions are shown in table (5.2), and it can be seen that no other change besides the upgrading to superfields has been made. This is because the charge configuration cancels chiral anomalies coming from fermions alone, as it happened in the previous version of this model.

In this thesis, only the Higgs sector is going to be dicussed. The fermion phenomenology has been already analysed in the paper of this model and further discussed by S. Alvarado and R. Martínez in [84,85]. However, some results from this part will be needed, because they give a preferred region for some VEVs in the model that makes FMH natural. At tree level, the SM quarks have the following approximate quadratic mass expressions:

$$\begin{aligned} m_d^2 &= 0, & m_u^2 &= 0, \\ m_s^2 &= 0, & m_c^2 &= \frac{v_1^2}{2}\frac{\left[h_{1u}^{22} g_{\chi' T} - h_{1T}^2 h_{\chi' u}^2\right]}{g_{\chi' T}^2 + h_{\chi' u}^2}, \\ m_b &= \frac{v_2'^2}{2}\left[(h_{2d}^{31})^2 + h_{2d}^{32})^2 + h_{2d}^{33})^2\right], & m_t &= \frac{v_1^2}{2}\left[(h_{1u}^{31})^2 + h_{1u}^{33})^2\right]. \end{aligned} \tag{5.1}$$

Here the subscripts u, d, s, c, b and t refere to up, down, strange, charm, bottom and top quarks, repectively. All the other couplings that have not been mentioned so far are

Table 5.1: Higgs content of the model, non-universal X quantum number, $\mathbb{Z}_2$ parity and hypercharge

Higgs Scalar Doublets	$X^{\pm}$	Y	Higgs Scalar Singlets	$X^{\pm}$	Y
$\hat{\Phi}_1 = \begin{pmatrix} \hat{\phi}_1^+ \\ \frac{\hat{h}_1+v_1+i\hat{\eta}_1}{\sqrt{2}} \end{pmatrix}$	$+\,2/3^+$	$+1$	$\hat{\chi} = \frac{\hat{\xi}_\chi+v_\chi+i\hat{\zeta}_\chi}{\sqrt{2}}$	$-\,1/3^+$	0
$\hat{\Phi}_2 = \begin{pmatrix} \hat{\phi}_2^+ \\ \frac{\hat{h}_2+v_2+i\hat{\eta}_2}{\sqrt{2}} \end{pmatrix}$	$+\,1/3^-$	$+1$	$\sigma = \frac{\hat{\sigma}_\chi+i\hat{\zeta}_\sigma}{\sqrt{2}}$	$-\,1/3^-$	0
$\hat{\Phi}'_1 = \begin{pmatrix} \frac{\hat{h}'_1+v'_1+i\hat{\eta}'_1}{\sqrt{2}} \\ \hat{\phi}_1^{-\prime} \end{pmatrix}$	$-\,2/3^+$	-1	$\hat{\chi}' = \frac{\hat{\xi}'_\chi+v'_\chi+i\hat{\zeta}'_\chi}{\sqrt{2}}$	$+\,1/3^+$	0
$\hat{\Phi}'_2 = \begin{pmatrix} \frac{\hat{h}'_2+v'_2+i\hat{\eta}'_2}{\sqrt{2}} \\ \hat{\phi}_2^{-\prime} \end{pmatrix}$	$-\,1/3^-$	-1	$\sigma' = \frac{\hat{\xi}'_\sigma+i\hat{\zeta}'_\sigma}{\sqrt{2}}$	$+\,1/3^-$	0

Table 5.2: Superfields related to the fermion content, non-universal X quantum number and parity $\mathbb{Z}_2$.

Left-Handed Fermions	$X^{\pm}$	Right-Handed Fermions	$X^{\pm}$
SM Quarks			
$\hat{q}_L^1 = \begin{pmatrix} \hat{u}^1 \\ \hat{d}^1 \end{pmatrix}_L$	$+\,1/3^+$	$\hat{u}_L^{1\,c}$	$-\,2/3^+$
		$\hat{u}_L^{2\,c}$	$-\,2/3^-$
$\hat{q}_L^2 = \begin{pmatrix} \hat{u}^2 \\ \hat{d}^2 \end{pmatrix}_L$	0^-	$\hat{u}_L^{3\,c}$	$-\,2/3^+$
		$\hat{d}_L^{1\,c}$	$+\,1/3^-$
$\hat{q}_L^3 = \begin{pmatrix} \hat{u}^3 \\ \hat{d}^3 \end{pmatrix}_L$	0^+	$\hat{d}_L^{2\,c}$	$+\,1/3^-$
		$\hat{d}_L^{3\,c}$	$+\,1/3^-$
SM Leptons			
$\hat{\ell}_L^e = \begin{pmatrix} \hat{\nu}^e \\ \hat{e}^e \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{e\,c}$	$-\,1/3^-$
		$\hat{\nu}_L^{\mu\,c}$	$-\,1/3^-$
$\hat{\ell}_L^\mu = \begin{pmatrix} \hat{\nu}^\mu \\ \hat{\mu}^\mu \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{\tau\,c}$	$-\,1/3^-$
		$\hat{e}_L^{e\,c}$	$+\,4/3^-$
$\hat{\ell}_L^\tau = \begin{pmatrix} \hat{\nu}^\tau \\ \hat{\tau}^\tau \end{pmatrix}_L$	-1^+	$\hat{e}_L^{\mu\,c}$	$+\,1/3^-$
		$\hat{e}_L^{\tau\,c}$	$+\,4/3^-$
Non-SM Quarks			
$\hat{\mathcal{T}}_L$	$+\,1/3^-$	$\hat{\mathcal{T}}_L^c$	$-\,2/3^-$
$\mathcal{J}_L^1$	0^+	$\hat{\mathcal{J}}_L^{c\,1}$	$+\,1/3^+$
$\mathcal{J}_L^2$	0^+	$\hat{\mathcal{J}}_L^{c\,2}$	$+\,1/3^+$
Non-SM Leptons			
$\hat{E}_L$	-1^+	$\hat{E}_L^c$	$+\,2/3^+$
$\hat{\mathcal{E}}_L$	$-\,2/3^+$	$\hat{\mathcal{E}}_L^c$	$+1^+$
Majorana Neutrinos		$\mathcal{N}_R^{1,2,3}$	0^-

Yukawa coupling constants. At tree level, the lightest quarks are massless, but this is corrected through radiative corrections with the singlet fields σ and σ'. For addresing a natural FMH, one must have that $v_1 \sim m_t$ and $v_2' \sim m_b$. The charm quark is naturally lighter than the top, due to the subtraction of Yukawa coupling constants. As for the SM charged leptons, the following squared mass expressions were obtained at tree level:

$$\begin{aligned} m_e^2 &= 0, & m_\mu^2 &= \frac{v_2'^2}{2}\left[(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2\right], \\ m_\tau^2 &= \frac{v_2'^2}{2}\left[(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2\right]. \end{aligned} \tag{5.2}$$

The subscripts e, μ and τ refer to the electron, the muon and the tauon, repectively. The electron gets is mass via radiative corrections with the fields σ and σ'. The condition $v_2' \sim m_\tau$ is consistent with the bottom mass, but the muon mass has to be fine tuned.

Flavor changing neutral currents?

When a model has more than one Higgs doublet, in general, it has flavor changes due to neutral currents. This is because the neutral current interactions cannot be diagonalized at the same time as the mass matrices. The flavor changing neutral currents (FCNC) is strongly constrained [86]. It is preferred that a model does not predict such a phenomenology, at least at tree level. In this case, there are 4 doublets, although only two are truly involved in the possible flavor change with neutral currents. This is because half of the doublets couples with up quarks only, and the other half exclusively with the down quarks. The possibility of FCNC may be exhibited by looking at the mass matrix of the up-quarks:

$$M_U = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & h_{2u}^{12}v_2 & 0 \\ 0 & h_{1u}^{22}v_1 & 0 \\ h_{1u}^{31}v_1 & 0 & h_{1u}^{33}v_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & h_{1u}^{22}v_1 & 0 \\ h_{1u}^{31}v_1 & 0 & h_{1u}^{33}v_1 \end{pmatrix} + \begin{pmatrix} 0 & h_{2u}^{12}v_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right], \tag{5.3}$$

where the matrix was separated into a matrix than only depends on the doublet ϕ_1 and another, which hangs on merely on ϕ_2. The source of the FCNC relies on the question if both matrices can be diagonalized simultaneously with the same unitary matrix. The MSSM does not have this problem, since its two doublets couple to up quarks or down quarks only, and this is called a type II two-higgs doublet model [87]. For avoiding that FCNC appear in the model, a first solution by hand would be to set arbitrarily the yukawa coupling $h_{2u}^{12} = 0$. This does not spoil the mass spectrum shown in (5.1). If this is the case, the model would be similar to a type II two higgs-doublet model, regarding the mass generation of quarks. The down quarks do not bring appearance of FCNC, since the mass matrix only depends on one doublet at tree level:

$$M_D = \frac{v_2'}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_{2d}^{31} & h_{2d}^{32} & h_{2d}^{33} \end{pmatrix} \tag{5.4}$$

Couplings with singlets could alter the tree level phenomenology, but considering that their corrections obey a see saw rotation, they are highly suppressed by VEVs like v_χ and v'_χ. A more detailed analysis considering $h^{12}_{2u} \neq 0$ is important for constraining the model to be in agreement with the FCNC non-detection.

5.2 Higgs sector

The most general renormalizable superpotential is made of neutrally charged terms respect to the symmetry group and involves all Higgs fields. It is given in the following equation:

$$W_\phi = -\mu_1 \hat{\Phi}'_1 \hat{\Phi}_1 - \mu_2 \hat{\Phi}'_2 \hat{\Phi}_2 - \mu_\chi \hat{\chi}' \hat{\chi} - \mu_\sigma \hat{\sigma}' \hat{\sigma} + \lambda_1 \hat{\Phi}'_1 \hat{\Phi}_2 \hat{\sigma}' + \lambda_2 \hat{\Phi}'_2 \hat{\Phi}_1 \sigma. \tag{5.5}$$

The first four terms are similar to the μ-term in the MSSM, having each coefficient with a mass dimension equal to one. The following two terms are trilinear terms of two doublets and a singlet. From the superpotential, the products of superfields were expanded and only the terms that remain after integration in the spinor coordinates were extracted according to the equations (4.48). After that, all F-terms were obtained by their equations of motion, all following the same form as in equation (4.50). Finally, the potential coming from F-terms is just $V_F = \sum F_i F_i^*$, where the index i runs over all superfields within the Higgs sector. It is explicitly given by:

$$\begin{aligned} V_F = {} & \mu_1^2(\Phi_1^\dagger \Phi_1 + \Phi_1'^\dagger \Phi'_1) + \mu_2^2(\Phi_2^\dagger \Phi_2 + \Phi_2'^\dagger \Phi'_2) + \mu_\chi^2(\chi^* \chi + \chi'^* \chi') + \mu_\sigma^2(\sigma^* \sigma + \sigma'^* \sigma') \\ & + \Big(\lambda_1^2 |\epsilon_{ij} \Phi_1'^i \Phi_2^j|^2 + \lambda_2^2 |\epsilon_{ij} \Phi_2'^i \Phi_1^j|^2 + \lambda_1^2(\Phi_2^\dagger \Phi_2 + \Phi_1'^\dagger \Phi'_1 \sigma'^* \sigma' + \lambda_2^2(\Phi_1^\dagger \Phi_1 + \Phi_2'^\dagger \Phi'_2)\sigma^* \sigma - \lambda_1 \mu_1 \Phi_1^\dagger \Phi_2 \sigma' \\ & - \lambda_1 \mu_2 \Phi_2'^\dagger \Phi'_1 \sigma' - \lambda_2 \mu_1 \Phi_1'^\dagger \Phi'_2 \sigma - \lambda_2 \mu_2 \Phi_2^\dagger \Phi_1 \sigma - \lambda_1 \mu_\sigma \epsilon_{ij} \Phi_1'^i \Phi_2^j - \lambda_2 \mu_\sigma \epsilon_{ij} \Phi_2'^i \Phi_1^j + h.c. \Big) \end{aligned} \tag{5.6}$$

To ensure the gauge symmetry of the model, we must also have the D-terms coming from the Käler potential. The local group directly involved in the Higgs sector is only $SU(2)_L \times U(1)_Y \times U(1)_X$. Thus, D-terms were found, according to the equation (4.51). Finally, the potential coming from D-terms is $V_D = \sum D_j D_j^*$, where the index j runs over the three symmetry groups directly involved in the Higgs sector. Being g_X the coupling constant of $U(1)_X$ and keeping the same notation of the coupling constants in the SM, the D-terms potential of the model is shown in the following lines,

$$\begin{aligned} V_D = {} & \frac{g^2}{2}\Big[|\Phi_1^\dagger \Phi_2|^2 + |\Phi_1'^\dagger \Phi'_2|^2 + |\Phi_1'^\dagger \Phi_1|^2 + |\Phi_1'^\dagger \Phi_2|^2 + |\Phi_2'^\dagger \Phi_1|^2 + |\Phi_2'^\dagger \Phi_2|^2 - |\Phi_1|^2 |\Phi_2|^2 - |\Phi'_1|^2 |\Phi'_2|^2 \Big] \\ & + \frac{g^2 + g'^2}{8}(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \Phi_1'^\dagger \Phi'_1 - \Phi_2'^\dagger \Phi'_2)^2 \\ & + \frac{g_X^2}{2}\left[\frac{2}{3}(\Phi_1^\dagger \Phi_1 - \Phi_1'^\dagger \Phi'_1) + \frac{1}{3}(\Phi_2^\dagger \Phi_2 - \Phi_2'^\dagger \Phi'_2) - \frac{1}{3}(\chi^* \chi - \chi'^* \chi') - \frac{1}{3}(\sigma^* \sigma - \sigma'^* \sigma') \right]^2 \end{aligned} \tag{5.7}$$

The non-universality of the additional Abelian $U(1)_X$ is exhibited in the last line, since every coefficient corresponds to the fields' charges under $U(1)_X$. Last but not least, a soft-supersymmetry breaking potential is put by hand for explaining why supersymmetry

has not been yet observed. However, gauge symmetry is still present in this potential.

$$
\begin{aligned}
-V_{soft} = & -m_1^2\Phi_1^\dagger\Phi_1 - m_1'^2\Phi_1'^\dagger\Phi_1' - m_2^2\Phi_2^\dagger\Phi_2 - m_2'^2\Phi_2'^\dagger\Phi_2' - m_\chi^2\chi^\dagger\chi - m_\chi'^2\chi'^\dagger\chi' - m_\sigma^2\sigma^\dagger\sigma - m_\sigma'^2\sigma'^\dagger\sigma' \\
& + \Big[\mu_{11}^2\epsilon_{ij}(\Phi_1'^i\Phi_1^j) + \mu_{22}^2\epsilon_{ij}(\Phi_2'^i\Phi_2^j) + \mu_{\chi\chi}^2(\chi\chi') + \mu_{\sigma\sigma}^2(\sigma\sigma') - \tilde{\lambda}_1\Phi_1'^\dagger\Phi_2\sigma' - \tilde{\lambda}_2\Phi_2'^\dagger\Phi_1\sigma \\
& + \frac{2\sqrt{2}}{9}(k_1\Phi_1^\dagger\Phi_2\chi' - k_2\Phi_1^\dagger\Phi_2\chi^* + k_3{\Phi_1'}^\dagger\Phi_2'\chi - k_4{\Phi_1'}^\dagger\Phi_2'\chi'^*) + h.c.\Big]
\end{aligned} \tag{5.8}
$$

The last terms, which have k_i as coupling constant, do not satisfy the parity symmetry that was supposedly imposed from the beginning. The reason for including these terms is that, if they were not there, some particles would remain massless. Terms like this were also included in the non-supersymmetric version of the model for the same reason. This will be understood later in this chapter. Important to note from the soft-supersymmetry breaking potential is that the terms in the first line give a difference in the mass between non-primed and primed fields, as it happened in the MSSM. Having all the components, the full potential of the scalar fields from the Higgs sector is just the addition of the three potentials,

$$V_H = V_F + V_D + V_{soft}. \tag{5.9}$$

The spontaneous symmetry breaking is mandatory for generating the masses of particles. In the model, the following vacuum expectation values are assumed for the Higgs doublets:

$$\hat{\Phi}_i|_{VEV} = \begin{pmatrix} 0 \\ \frac{v_i}{\sqrt{2}} \end{pmatrix} \qquad \hat{\Phi}_i'|_{VEV} = \begin{pmatrix} \frac{v_i'}{\sqrt{2}} \\ 0 \end{pmatrix} \tag{5.10}$$

$$\hat{\chi}|_{VEV} = \frac{v_\chi}{\sqrt{2}} \qquad \hat{\chi}'|_{VEV} = \frac{v_\chi'}{\sqrt{2}} \tag{5.11}$$

$$\hat{\sigma}|_{VEV} = 0 \qquad \hat{\sigma}'|_{VEV} = 0 \tag{5.12}$$

In order to have a potential minimum, one can establish the condition $\frac{\partial V_H}{\partial v_i} = 0$ for all VEVs. Defining for the sake of having shorter equations $m_{Hk}^2 = m_k^2 + \mu_k^2$ and $m_{Hk}'^2 = m_k'^2 + \mu_k'^2$, where k runs over all the fields in the Higgs sector, the minimum conditions are shown in the following lines:

$$
\begin{aligned}
m_{H1}^2 + \frac{1}{8}(g^2+g'^2)C_{EW} + \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1'}{v_1} + \frac{\lambda_2^2}{2}v_2'^2 = 0 \\
m_{H1}'^2 - \frac{1}{8}(g^2+g'^2)C_{EW} - \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1}{v_1'} + \frac{\lambda_1^2}{2}v_1'^2 = 0 \\
m_{H2}^2 + \frac{1}{8}(g^2+g'^2)C_{EW} + \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2'}{v_2} + \frac{\lambda_1^2}{2}v_2^2 = 0 \\
m_{H2}'^2 - \frac{1}{8}(g^2+g'^2)C_{EW} - \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2}{v_2'} + \frac{\lambda_2^2}{2}v_1^2 = 0 \\
M_\chi^2 - \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi'}{v_\chi} = 0 \\
M_\chi'^2 + \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi}{v_\chi'} = 0,
\end{aligned} \tag{5.13}
$$

with $C_{EW} = v_1^2 + v_2^2 - v_1'^2 - v_2'^2$ and $C_X = 2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 + v_\chi'^2 - v_\chi^2$. Equations (5.13) are paramount for studying the mass spectrum for scalar particles in the Higgs sector.

5.2.1 Charged spectrum

The full scalar potential of the Higgs sector (5.9) has to be further expanded by doing the inner product of doublets. From there, mass terms for the fields were extracted and the minimum conditions (5.13) were used. The mass terms of the charged bosons can be arranged in a matrix form like $V_C^\dagger \mathcal{M}_C V_C$, where $V_C \equiv (\phi_1^+, \phi_2^+, \phi_1'^+, \phi_2'^+)$. Before showing the mass matrix $\mathcal{M}_C$ explicitly, it is important to define the following expressions that will be used all over the discussion for having shorter expressions:

$$\begin{aligned}
f_{ng} &= \frac{g^2 + g'^2}{8} + \frac{n}{18} g_X^2 \quad \text{with n an integer} \\
f_{1k} &= k_2 v_\chi - k_1 v_\chi' \\
f_{2k} &= k_4 v_\chi' - k_3 v_\chi \\
k_{23} &= k_2 v_1 v_2 - k_3 v_1' v_2' \\
k_{14} &= k_4 v_1' v_2' - k_1 v_1' v_2' \\
M_\sigma &= \frac{1}{2}(\mu_\sigma^2 + m_\sigma^2) - \frac{g_X}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2) \\
M_\sigma' &= \frac{1}{2}(\mu_\sigma^2 + m_\sigma'^2) - \frac{g_X}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2)
\end{aligned} \tag{5.14}$$

The 4×4 mass matrix of the charged scalar bosons, from which the eigenvalues are the masses of the particles, is given right ahead:

$$\frac{1}{2}\mathcal{M}_C = \begin{pmatrix} M_C & m_C \\ m_C^T & M_C' \end{pmatrix}, \tag{5.15}$$

where the matrix is defined in blocks, because it is too long to show here. Before showing the blocks, the following expressions have been also defined for the sake of simplicity:

$$\begin{aligned}
m_{11}^C &= \frac{g^2}{4}(v_1'^2 + v_2'^2 - v_2^2) - \frac{2v_2}{9v_1} f_{1k}, \qquad m_{11}^{C\prime} = \frac{g^2}{4}(v_1'^2 + v_2^2 - v_2'^2) - \frac{2v_2'}{9v_1'} f_{2k}, \\
m_{22}^C &= \frac{g^2}{4}(v_1'^2 + v_2'^2 - v_1^2) - \frac{2v_1}{9v_2} f_{1k}, \qquad m_{22}^{C\prime} = \frac{g^2}{4}(v_1^2 + v_2^2 - v_1'^2) - \frac{2v_1'}{9v_2'} f_{2k}.
\end{aligned} \tag{5.16}$$

The part corresponding to the non-primed charged fields is given by M_C,

$$M_C = \begin{pmatrix} m_{11}^C + \frac{v_1'}{v_1}\mu_{11}^2 - \frac{1}{2}\lambda_2^2 v_2'^2 & \frac{g^2}{4} v_1 v_1' + \mu_{11}^2 \\ \frac{g^2}{4} v_1 v_1' + \mu_{11}^2 & m_{11}^{C\prime} + \frac{v_1}{v_1'}\mu_{11}^2 - \frac{1}{2}\lambda_1^2 v_2^2 \end{pmatrix}. \tag{5.17}$$

The block corresponding to mass terms mixing primed with non-primed fields is m_c, which

is given right ahead,

$$m_C = \begin{pmatrix} \frac{g^2}{4}v_1v_2 + \frac{2}{9}f_{1k} & \frac{g^2}{4}v_1v_2' - \frac{1}{2}\lambda_2^2 v_1 v_2' \\ \frac{g^2}{4}v_1'v_2 - \frac{1}{2}\lambda_1^2 v_2 v_1' & \frac{g^2}{4}v_1'v_2' + \frac{2}{9}f_{2k} \end{pmatrix}. \tag{5.18}$$

Finally, the matrix regarding to the mixing between only primed fields is given by:

$$M_C' = \begin{pmatrix} m_{22}^C + \frac{v_2'}{v_2}\mu_{22}^2 - \frac{1}{2}\lambda_1^2 v_1'^2 & \frac{g^2}{4}v_2v_2' + \mu_{22}^2 \\ \frac{g^2}{4}v_2v_2' + \mu_{22}^2 & m_{22}^{C\prime} + \frac{v_2}{v_2'}\mu_{22}^2 - \frac{1}{2}\lambda_2^2 v_1^2 \end{pmatrix}. \tag{5.19}$$

The rank of the previous matrix is equal to 3. This means that there is an eigenvalue equal to 0, corresponding to the Goldstone boson that gives mass to the W boson. Using *Mathematica 11* for finding approximated analytical expressions, the following eigenvalues were obtained:

$$\begin{aligned} m_{G_W^\pm}^2 &= 0, \\ m_{H_1^\pm}^2 &\approx \frac{2v^2(v_1v_2(k_1v_\chi' - k_2v_\chi) + v_1'v_2'(k_3v_\chi - k_4v_\chi'))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)}, \\ m_{H_2^\pm}^2 &\approx \mu_{11}^2\frac{v_1^2 + v_1'^2}{v_1v_1'}, \\ m_{H_3^\pm}^2 &\approx \mu_{22}^2\frac{v_2^2 + v_2'^2}{v_2v_2'}. \end{aligned} \tag{5.20}$$

The approximations were done by making the set of electroweak VEV very small compared to the scale of $U(1)_X$, which is exhibited in the VEVs v_χ and v_χ'. Not only that, but also small compared to the scale of soft supersymmetry breaking. Thereby, Taylor expansions over ratios between electroweak VEVs and other mass parameters, for instance v_2/μ_{11}, were dropped after the third order. There are 3 massive charged scalar eigenstates $H_1^\pm$, $H_2^\pm$ and $H_3^\pm$, whose masses depend on the free parameters $k_1, k_2, k_3, k_4, \mu_{11}, \mu_{22}, v_\chi, v_\chi'$. The need of the parity breaking terms with coupling constants k_i can be seen in the mass expression of the charged H_1. If such parity terms were not considered, then there would be another massless state. However, only one of them is the Goldstone boson, and for this reason, there would be an observable massless charged particle. Such an state has not been detected. The heaviness of the H_1, compared to the electroweak scale, may be explained by the VEVs v_χ and v_χ'. The coefficients k_1, k_2, k_3 and k_4 should be big enough to explain the heaviness of H_1. Explicitly, the following inequations are required:

$$k_iv_j \gg g_X^2 v_\chi v_j, g_X^2 v_\chi' v_j, g_X^2 v_\chi v_j', g_X^2 v_\chi' v_j' \quad \text{for i,j=1,2.} \tag{5.21}$$

The k parameters shall also be tuned to avoid cancellations that spoil the heaviness of the H_1 boson. The other massive states, H_2 and H_3, lie at roughly the scale of supersymmetry breaking. Indeed, taking the limit of all electroweak VEV approaching to zero, we get that:

$$m^2_{H_2^\pm} \to \mu^2_{11} \qquad\qquad m^2_{H_3^\pm} \to \mu^2_{22} \tag{5.22}$$

5.2.2 Pseudoscalar spectrum

The mass terms for the pseudoscalar particles, which come from the imaginary part of the neutral component of doublets and the imaginary part of singlets, can be also expressed in a matrix form like $V_\eta^\dagger \mathcal{M}_\eta V_\eta$. Having the basis $V_\eta = (\eta_1, \eta_1', \eta_2, \eta_2', \zeta_\chi, \zeta_\chi', \zeta_\sigma, \zeta_\sigma)$ and using the minima conditions (5.13), the mass matrix for CP-odd particles turns out to be the following 8×8 matrix.

$$\frac{1}{2}\mathcal{M}_\eta = \begin{pmatrix} \mathcal{M}_{\eta\eta} & \mathcal{M}_{\eta\zeta} \\ \mathcal{M}^T_{\eta\zeta} & \mathcal{M}_{\zeta\zeta} \end{pmatrix}. \tag{5.23}$$

The matrix has been shown in block form, where $\mathcal{M}_{\eta\eta}$ regards the couplings between CP-odd particles coming from doublets only, $\mathcal{M}_{\eta\zeta}$ the couplings between pseudoscalars coming from doublets and singlets, and $\mathcal{M}_{\zeta\zeta}$ the couplings between fields coming from singlets only. The matrices conforming the mass matrix of the CP-odd scalar particles are given in the following equations:

$$\mathcal{M}_{\eta\eta} = \begin{pmatrix} \frac{\mu^2_{11}v_1'}{2v_1} - \frac{f_{1k}v_2}{9v_1} & \frac{\mu^2_{11}}{2} & \frac{f_{1k}}{9} & 0 \\ * & \frac{\mu^2_{11}v_1}{2v_1'} - \frac{f_{2k}v_2'}{9v_1'} & 0 & \frac{f_{2k}}{9} \\ * & * & \frac{\mu^2_{22}v_2'}{2v_2} - \frac{f_{1k}v_1}{9v_2} & \frac{\mu^2_{22}}{2} \\ * & * & * & \frac{\mu^2_{22}v_2}{2v_2'} - \frac{f_{2k}v_1'}{9v_2'} \end{pmatrix}. \tag{5.24}$$

$$\mathcal{M}_{\eta\zeta} = \frac{1}{9}\begin{pmatrix} -k_2v_2 & -k_1v_2 & \frac{1}{2\sqrt{2}}(\lambda_2\mu_2v_2 - \tilde{\lambda}_2v_2') & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_1v_2 + \lambda_2\mu_\sigma v_2') \\ -k_3v_2' & -k_4v_2' & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_1v_2' + \lambda_1\mu_\sigma v_2) & \frac{1}{2\sqrt{2}}(\lambda_1\mu_2v_2' - \tilde{\lambda}_1v_2) \\ k_2v_1 & k_1v_1 & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_2v_1 + \lambda_1\mu_\sigma v_1') & \frac{1}{2\sqrt{2}}(\lambda_1\mu_1v_1 - \tilde{\lambda}_1v_1') \\ k_3v_1' & k_4v_1' & \frac{1}{2\sqrt{2}}(\lambda_2\mu_1v_1' - \tilde{\lambda}_2v_1) & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_2v_1' + \lambda_2\mu_\sigma v_1) \end{pmatrix}. \tag{5.25}$$

$$\mathcal{M}_{\zeta\zeta} = \begin{pmatrix} \frac{v_\chi'\mu^2_{\chi\chi}}{2v_\chi} - \frac{k_{23}}{9v_\chi} & \frac{\mu^2_{\chi\chi}}{2} & 0 & 0 \\ * & \frac{v_\chi\mu^2_{r2}}{2v_\chi'} - \frac{k_{14}}{9v_\chi'} & 0 & 0 \\ * & * & M_\sigma^2 + \frac{\lambda_2^2}{4}(v_1^2 + v_2'^2) & \frac{\mu_{\sigma\sigma}}{2} \\ * & * & * & M_\sigma'^2 + \frac{\lambda_1^2}{4}(v_2^2 + v_1'^2) \end{pmatrix}. \tag{5.26}$$

The rank of the 8×8 matrix $\mathcal{M}_\eta$ is 6. Thereby, there are two massless eigenstates that correspond to the Goldstone bosons that give masses to the Z and Z' bosons. For finding expressions for the finite mass eigenvalues, a perturbation theory had to be done. A

similar procedure to the seesaw mechanism in neutrinos was performed. A scale hierarchy between matrix entries was assumed for doing this diagonalization method:

$$\mathcal{O}(\mathcal{M}_{\zeta\zeta}) > \mathcal{O}(\mathcal{M}_{\eta\zeta}), \mathcal{O}(\mathcal{M}_{\eta\eta}). \tag{5.27}$$

The assumption relies in the fact that soft-supersymmetry and $U(1)_X$ breaking scales should lie above the electroweak scale, which is of course not yet reached by colliders. Although this means that the mass parameters should obey similar inequalities, it must not be forgotten that SUSY-breaking mass parameters like μ_{11} and μ_{22} should lie at scales beyond the TeV. Having this in mind, following the equation (5.27), and taking into account the equation (5.21), we have the following hierarchy of the mass parameters:

$$\mu_{\sigma\sigma}, \mu_{\chi\chi}, M_\sigma, M'_\sigma \gg \mu_{11}, \mu_{22} \gg k_i v_j \gg g_X^2 v_\chi v_j, g_X^2 v'_\chi v_j, g_X^2 v_\chi v'_j, g_X^2 v'_\chi v'_j \quad \text{for i,j=1,2} \tag{5.28}$$

Thereby, the following block diagonal form of the mass matrix can be obtained,

$$\tilde{\mathcal{M}}_\eta \approx \begin{pmatrix} \tilde{\mathcal{M}}_{\eta\eta} & 0 \\ 0 & \mathcal{M}_{\zeta\zeta} \end{pmatrix}, \tag{5.29}$$

$$\text{where } \tilde{\mathcal{M}}_{\eta\eta} = \mathcal{M}_{\eta\eta} - \mathcal{M}_{\eta\zeta}\mathcal{M}_{\zeta\zeta}^{-1}\mathcal{M}_{\eta\zeta}^T. \tag{5.30}$$

The next step is to diagonalize the 4×4 matrix $\mathcal{M}_{\zeta\zeta}$ and the 4×4 matrix $\tilde{\mathcal{M}}_{\eta\eta}$ separately. For $\mathcal{M}_{\zeta\zeta}$, the limit of the coefficients k_i approaching to zero was taken, with $i = 1, 2, 3, 4$. This is totally consistent with the inequalities (5.27). Thus, approximate expressions of mass eigenstates could be obtained with *Mathematica 11*, and they are given by:

$$m_{\eta 5}^2 = 0, \qquad\qquad m_{\eta 6}^2 \approx \mu_{\chi\chi}^2 \frac{v_\chi^2 + {v'_\chi}^2}{v_\chi v'_\chi}. \tag{5.31}$$

$$\begin{aligned} m_{\eta 7}^2 &= (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] \\ &\quad - \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2}, \\ m_{\eta 8}^2 &= (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] \\ &\quad + \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2}. \end{aligned}$$

There is one Goldstone boson G_W that gives mass to the Z', since it comes from the singlet sector. The other massive eigenvalues depend on the set of free parameters related to singlets and trilinear couplings. The heaviness of the CP-odd η_6 is ensured by the mass parameter $\mu_{\chi\chi}$ from the soft-supersymmetry breaking potential. The other states, η_8 and η_9, can be heavy thanks to the parameters $\mu_{\sigma\sigma}$, m_σ and m'_σ. Therefore, the fields $\hat{\sigma}$ and $\hat{\sigma}'$ are also necessary for making η_7 and η_8 naturally heavy. After the seesaw mechanism applied in equation (5.30), the matrix corresponding to the lightest states is

too complicated to show here. Its rank happens to be 3, and thereby there is another goldstone boson η^1 that gives mass to the Z boson. Therefore, it is massless,

$$m^2_{\eta 1} = 0. \tag{5.32}$$

A simplification can be made for obtaining the mass of the biggest eigenvalues of the matrix given in equation (5.30). Since those are supposed to be at an energy scale higher than the ones reached so far, then the electroweak VEV can be considered small. Taking the limit of electroweak VEVs approaching to 0, except on ratios between such VEV like v_1/v_2, the following block diagonalized matrix is obtained:

$$\tilde{\mathcal{M}}_{\eta\eta}(v_i, v_i' \to 0) = \begin{pmatrix} \frac{\mu_{11}^2}{2}\frac{v_1'}{v_1} & \frac{\mu_{11}^2}{2} & 0 & 0 \\ * & \frac{\mu_{11}^2}{2}\frac{v_1}{v_1'} & 0 & 0 \\ * & * & \frac{\mu_{22}^2}{2}\frac{v_2'}{v_2} & \frac{\mu_{22}^2}{2} \\ * & * & * & \frac{\mu_{22}^2}{2}\frac{v_2}{v_2'} \end{pmatrix} \tag{5.33}$$

Using this approximation, 2 eigenstates are massless and 2 are massive. The two massive states are given by:

$$m^2_{\eta 3} \approx \mu_{11}^2 \frac{v_1^2 + v_1'^2}{v_1 v_1'}, \qquad m^2_{\eta 4} \approx \mu_{22}^2 \frac{v_2^2 + v_2'^2}{v_2 v_2'}. \tag{5.34}$$

For obtaining the lightest, but not massless, eigenvalue of the pseudoscalars, the Ferrari method [88] was used for solving the characteristic equation. The solutions of a quartic equation are shown in the appendix B. Since one of the solutions is zero, the solutions of the cubic equation derived in the method were taken. The solution, that does not correspond to one of the expressions in (5.34) when taking the limit of small electroweak VEVs, was the one corresponding to the lightest CP-odd particle. Considering the parameter hierarchy shown in equation (5.28) and dropping negligible terms, the mass of the lightest pseudoscalar is:

$$m^2_{\eta 2} \approx \frac{2v^2(v_1 v_2(k_1 v_\chi' - k_2 v_\chi) + v_1' v_2'(k_3 v_\chi - k_4 v_\chi'))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)}. \tag{5.35}$$

Similar to the lightest charged scalar boson, whose mass is shown in the second equation of (5.20), the parameters k_1, k_2, k_3 and k_4 should be also tuned for avoiding cancellations. However, the tuning that has to be done is consistent in both equations.

5.2.3 Scalar spectrum

The scalar particles come from the real part of the neutrally charged fields that appear in doublets and singlets. The mass terms were put in the matrix form $V_h^\dagger M_h V_h$, with $V_h = (h_1, h_1', h_2, h_2', \xi_\chi, \xi_\chi', \xi_\sigma, \xi_\sigma')$. M_h is an 8×8 matrix that has rank 8. That means that all scalar fields are massive. The lightest eigenvalue should correspond naturally to the 125 GeV Higgs boson. The matrix M_h is shown below,

$$\frac{1}{2}M_h = \begin{pmatrix} M_{hh} & M_{h\xi} \\ M_{h\xi}^T & M_{\xi\xi} \end{pmatrix}, \tag{5.36}$$

where a block form has been used. M_{hh} is a 4×4 matrix that collects mass couplings only between scalar fields coming from doublet. It is given right ahead,

$$M_{hh} = \begin{pmatrix} f_{4g}v_1^2 - \frac{v_2 f_{1k}}{9v_1} + \frac{v_1' \mu_{11}^2}{2v_1} & -f_{4g}v_1 v_1' - \frac{\mu_{11}^2}{2} & f_{2g}v_1 v_2 + \frac{f_{1k}}{9} & -f_{2g}v_1 v_2' + \frac{1}{2}\lambda_2^2 v_1 v_2' \\ * & f_{4g}{v_1'}^2 - \frac{v_2' f_{2k}}{9v_1'} + \frac{v_1 \mu_{11}^2}{2v_1'} & -f_{2g}v_1' v_2 + \frac{1}{2}\lambda_1^2 v_2 v_1' & f_{2g}v_1' v_2' + \frac{f_{2k}}{9} \\ * & * & f_{1g}v_2^2 - \frac{v_1 f_{1k}}{9v_2} + \frac{v_2' \mu_{22}^2}{2v_2} & -f_{1g}v_2 v_2' - \frac{\mu_{22}^2}{2} \\ * & * & * & f_{1g}{v_2'}^2 - \frac{v_1' f_{2k}}{9v_2'} + \frac{v_2 \mu_{22}^2}{2v_2'} \end{pmatrix}. \tag{5.37}$$

The blocks outside the diagonal are 4×4 and regard mass terms between fields coming from singlets and doublets. $M_{h\xi}$ is given by:

$$M_{h\xi} = \begin{pmatrix} \frac{1}{9}(k_2 v_2 - g_X^2 v_1 v_\chi) & \frac{1}{9}(-k_1 v_2 + g_X^2 v_1 v_\chi') & \frac{1}{2\sqrt{2}}(\tilde{\lambda_2} v_2' - \lambda_2 \mu_2 v_2) & -\frac{1}{2\sqrt{2}}(\lambda_1 \mu_1 v_2 + \lambda_2 \mu_\sigma v_2') \\ \frac{1}{9}(-k_3 v_2' + g_X^2 v_1' v_\chi) & \frac{1}{9}(k_4 v_2' - g_X^2 v_1' v_\chi') - & -\frac{1}{2\sqrt{2}}(\lambda_2 \mu_1 v_2' + \lambda_1 \mu_\sigma v_2) & \frac{1}{2\sqrt{2}}(\tilde{\lambda_1} v_2 - \lambda_1 \mu_2 v_2') \\ \frac{1}{9}(k_2 v_1 - \frac{1}{2} g_X^2 v_2 v_\chi) & \frac{1}{9}(-k_1 v_1 + \frac{1}{2} g_X^2 v_2 v_\chi') & -\frac{1}{2\sqrt{2}}(\lambda_2 \mu_2 v_1 + \lambda_1 \mu_\sigma v_1') & \frac{1}{2\sqrt{2}}(\tilde{\lambda_1} v_1' - \lambda_1 \mu_1 v_1) \\ \frac{1}{9}(-k_3 v_1' + \frac{1}{2} g_X^2 v_2' v_\chi) & \frac{1}{9}(k_4 v_1' - \frac{1}{2} g_X^2 v_2' v_\chi') & \frac{1}{2\sqrt{2}}(\tilde{\lambda_2} v_1 - \lambda_2 \mu_1 v_1') & -\frac{1}{2\sqrt{2}}(\lambda_1 \mu_2 v_1' + \lambda_2 \mu_\sigma v_1) \end{pmatrix}. \tag{5.38}$$

Last but not least, the 4×4 matrix $M_{\xi\xi}$ only accounts for terms with singlets, which is shown in the next equation,

$$M_{\xi\xi} = \begin{pmatrix} \frac{g_X^2}{18} v_\chi^2 + \frac{v_\chi' \mu_{\chi\chi}^2}{2v_\chi} - \frac{k_{23}}{9v_\chi} & -\frac{g_X^2}{18} v_\chi v_\chi' - \frac{\mu_{\chi\chi}^2}{2} & 0 & 0 \\ * & \frac{g_X^2}{18} {v_\chi'}^2 + \frac{v_\chi \mu_{\chi\chi}^2}{2v_\chi'} - \frac{k_{14}}{9v_\chi'} & 0 & 0 \\ * & * & M_\sigma^2 + \frac{\lambda_2^2}{4}(v_1^2 + v_2'^2) & -\frac{\mu_{\sigma\sigma}}{2} \\ * & * & * & M_\sigma'^2 + \frac{\lambda_1^2}{4}(v_2^2 + v_1'^2) \end{pmatrix} \tag{5.39}$$

The parameter hierarchy in equation (5.28) makes possible the seesaw rotation of the matrix, because the following is implied:

$$\mathcal{O}(M_{\xi\xi}) > \mathcal{O}(M_{h\xi}), \mathcal{O}(M_{hh}). \tag{5.40}$$

Therefore, the following block diagonal form is obtained,

$$\begin{aligned}\tilde{M}_h &\approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix}, \\ \tilde{M}_{hh} &= M_{hh} - M_{h\xi} M_{\xi\xi}^{-1} M_{h\xi}^T \\ &\approx M_{hh}. \end{aligned} \tag{5.41}$$

The contributions from the seesaw rotation to the matrix M_{hh}, $M_{h\xi} M_{\xi\xi}^{-1} M_{h\xi}^T$, can be neglected. That is the reason why $\tilde{M}_{hh} \approx M_{hh}$ was supposed. The diagonalization of the matrix that has the greatest eigenvalues $M_{\xi\xi}$ is direct, since the fourth grade characteristic equation can be decoupled into two idependent quadratic equations. Thereby, the mass of the heaviest CP-even scalar particles is show right ahead,

$$\begin{aligned} m_{h5}^2 &\approx \frac{g_X^2}{9}(v_\chi^2 + {v'_\chi}^2) - \frac{2}{9}\frac{v_1 v_2 (k_2 v'_\chi - k_1 v_\chi) + v'_1 v'_2 (k_4 v_\chi - k_3 v'_\chi)}{v_\chi v'_\chi}, \\ m_{h6}^2 &\approx \mu_{\chi\chi}^2 \frac{v_\chi^2 + {v'_\chi}^2}{v_\chi v'_\chi}, \end{aligned} \tag{5.42}$$

$$\begin{aligned} m_{h7}^2 &\approx (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] \\ &\quad - \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2}, \\ m_{h8}^2 &\approx (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] \\ &\quad + \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2} \end{aligned} \tag{5.43}$$

The scalar bosons h_7 and h_8 depend mostly on parameters coming from couplings with the σ and σ' singlets. Since those singlets do not acquire VEV, they do not have any minima condition to fulfill. Therefore, their masses may be apparently set to an arbitrary high energy. The boson h_6 is naturally heavy mainly due to the parameter $\mu_{\chi\chi}$. As for the scalar particle h_5, there seems to be cancellation amongst terms. However, the second term is suppressed due to the $U(1)_X$ VEVs v_χ and v'_χ. For getting the mass expressions of the lightest CP-even particles in the model, a similar approach to the one followed in the discussion of the pseudoscalars can be used. In that way, the greatest eigenvalues of the matrix M_{hh} can be approximately found by taking the limit of small electroweak VEV. In this limit, the following matrix is obtained:

$$M_{hh}(v_i, v'_i \to 0) = \begin{pmatrix} \frac{\mu_{11}^2}{2}\frac{v'_1}{v_1} & -\frac{\mu_{11}^2}{2} & 0 & 0 \\ * & \frac{\mu_{11}^2}{2}\frac{v_1}{v'_1} & 0 & 0 \\ * & * & \frac{\mu_{22}^2}{2}\frac{v'_2}{v_2} & -\frac{\mu_{22}^2}{2} \\ * & * & * & \frac{\mu_{22}^2}{2}\frac{v_2}{v'_2} \end{pmatrix} \tag{5.44}$$

In this approximated form, there are two null eigenvalues. The other two corresponds to the greatest eigenvalues in this matrix, which turn out to be:

$$m_{h3}^2 \approx \mu_{11}^2 \frac{v_1^2 + v_1'^2}{v_1 v_1'}, \qquad\qquad m_{h4}^2 \approx \mu_{22}^2 \frac{v_2^2 + v_2'^2}{v_2 v_2'}. \tag{5.45}$$

Their heaviness is guaranteed by the mass parameters μ_{11} and μ_{22}. Using the Ferrari method for analytically solve the fourth grade characteristic equation, a third eigenvalue can be found:

$$m_{h2}^2 \approx \frac{2v^2(v_1 v_2(k_1 v_\chi' - k_2 v_\chi) + v_1' v_2'(k_3 v_\chi - k_4 v_\chi'))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)}. \tag{5.46}$$

Its heaviness can be ensured by tuning the parameters k_1, k_2, k_3 and k_4. This is consistent with the equations (5.20) and (5.35). With Ferrari method the lightest eigenvalue was also obtained, but its expression is too complicated to show. A different way was used for obtaining a shorter expression. First, the determinant of the matrix M_{hh} was found, because it is equal to the multiplication of the four eigenvalues. Negligible terms were dropped and only terms proportional to $\mu_{11}^2 \mu_{22}^2$ were considered,

$$\begin{aligned} Det(\tilde{M}_{hh}) \approx \frac{\mu_{11}^2 \mu_{22}^2}{2592} \Bigg[& (k_3 v_\chi - k_4 v_\chi') \Bigg((9(g^2 + g'^2) + 16 g_X^2) \frac{(v_1^2 - v_1'^2)^2}{v_1 v_2} + (9(g^2 + g'^2) + 4 g_X^2) \frac{(v_2^2 - v_2'^2)^2}{v_1 v_2} \\ & + 2(9(g^2 + g'^2) + 8 g_X^2) \frac{(v_1^2 - v_1'^2)(v_2^2 - v_2'^2)}{v_1 v_2} \Bigg) + (k_1 v_\chi' - k_2 v_\chi) \Bigg((9(g^2 + g'^2) + 16 g_X^2) \frac{(v_1^2 - v_1'^2)^2}{v_1' v_2'} \\ & + (9(g^2 + g'^2) + 4 g_X^2) \frac{(v_2^2 - v_2'^2)^2}{v_1' v_2'} + 2(9(g^2 + g'^2) + 8 g_X^2) \frac{(v_1^2 - v_1'^2)(v_2^2 - v_2'^2)}{v_1' v_2'} \Bigg) \Bigg]. \end{aligned} \tag{5.47}$$

Dividing this by the eigenvalues shown in (5.45) and (5.46), the approximated expression for the possible Higgs boson is:

$$m_{h1}^2 \approx \frac{g_X^2 (2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2)^2}{9(v_1^2 + v_2^2 + v_1'^2 + v_2'^2)} + \frac{(g^2 + g'^2)(v_1^2 + v_2^2 - v_1'^2 - v_2'^2)^2}{4(v_1^2 + v_2^2 + v_1'^2 + v_2'^2)} \tag{5.48}$$

In a similar way that it is done in the MSSM, ratios between electroweak VEVs can define angles. Thereby, defining $\tan^2 \tilde{\beta} = \frac{v_1^2 + v_2^2}{v_1'^2} + v_2'^2$, $\tan \beta_1 = \frac{v_1}{v_1'}$ and $\tan \beta_2 = \frac{v_2}{v_2'}$, the next equation is obtained:

$$\begin{aligned} m_{h1}^2 &= m_Z^2 \left(cos^2 2\tilde{\beta} + \frac{4}{9} \frac{g_X^2}{g^2 + g'^2} (cos 2\beta_1 + \cos 2\beta_2)^2 \right) \\ &\approx m_Z^2 \cos^2 2\tilde{\beta} + \Delta m_h^2 \end{aligned} \tag{5.49}$$

where the expression of the mass of the Z boson in the SM, shown in equation (3.18, was used. The lightest CP-even particle lies naturally at the electroweak scale, and because of that, it is the only candidate to be the 125 GeV Higgs boson.

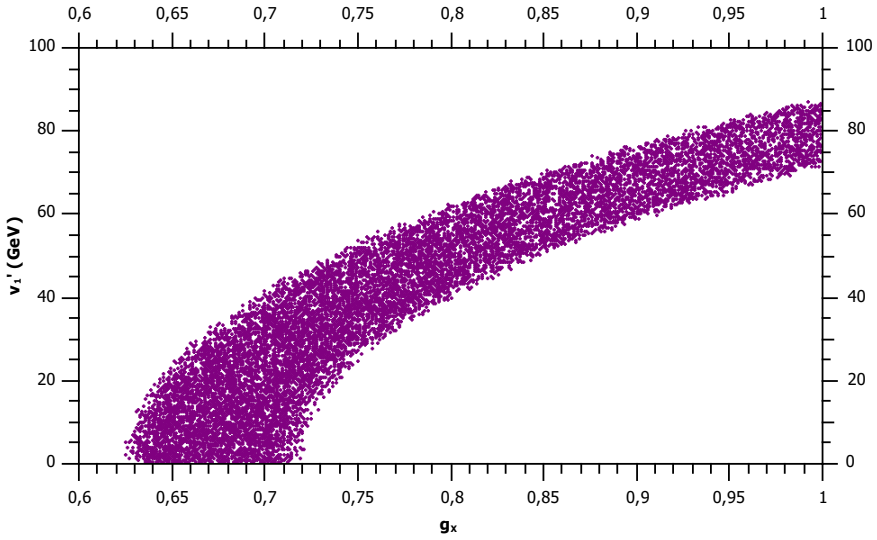

Figure 5.1: Region in the parameter space v_1' vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

The obtention of a 125 GeV Higgs boson

The expression (5.49) has been put in the same form as the 1-loop corrected mass of the Higgs boson in the MSSM, which is shown in equation (4.68). However, the Higgs mass obtained in this model is found at tree level. Here, the contribution Δm_h^2 comes from the D-term of the $U(1)_X$ gauge symmetry. Such a contribution can be naturally at the same order as the first term, $m_Z^2 \cos^2 2\tilde{\beta}$. Thus, compared to the MSSM, the obtention of a 125 GeV scalar boson is more natural. To see this, an exploration in the parameter spaces v_1' vs g_X and v_2 vs g_X was made, using the Montecarlo method for generating randomly the values of the parameters v_2, v_1', v_2' and g_X. Since v_1 and v_2' are restricted for giving a natural FMH, they were randomly spawned in the intervals [170 GeV,200 GeV] and [3 GeV,7 GeV], respectively. This can be checked in equations (5.1) and (5.2.) The VEV v_2 is not directly constrained by FMH, so it could run freely in the interval $0 < v_2 < 246$ GeV $= v$. Finally, the parameter v_1 was constrained by the following condition:

$$v = \sqrt{v_1^2 + v_1'^2 + v_2^2 + v_2'^2} = 246 \text{ GeV} \tag{5.50}$$

The necessity of the condition (5.50) is to get the right masses of the interaction bosons of the SM, but this will be seen later. The coupling constant of the $U(1)_X$ was explored in the interval [0,1], because we wanted the interactions from the extended group to be suitable for a perturbative treatment in possible future works. The points in the parameter space v_1' vs g_X that are in agreement with a Higgs mass of 125.3 ± 0.4 GeV at a 95% of confidence level are shown in the figure (5.1). The values for $g_X < 0.62$ happened to be excluded in the montecarlo exploration. Another important consequence from figure (5.1) was that $v_1' < 90$ GeV. Thereby, $v_1' = 0$ is allowed. In this case, there is an inert CP-even particle that is said to be inert, and therefore, it could be a candidate for dark matter [89]. However, this is not further studied in this work. The other plot v_2 vs g_X is shown in the figure (5.2). As well as for the last plot, only points in agreement with a Higgs mass of 125.3 ± 0.4 (GeV) at a 95% of confidence level are shown. Consistently

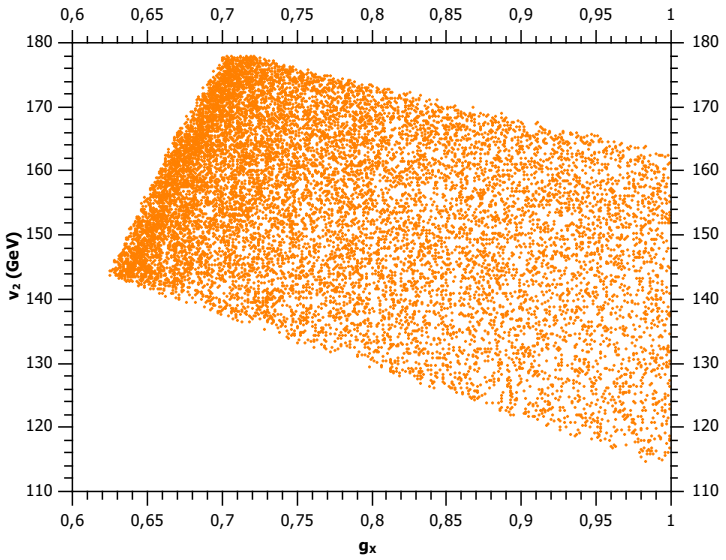

Figure 5.2: Region in the parameter space v_2 vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

with the plot (5.1), $g_X > 0.63$. The second VEV that was not constrained by FMH, v_2, falls in the interval 110 GeV $< v_2 <$ 180 GeV. In the expression of the Higgs mass in this model (5.49), there is a direct dependence on $\cos(2\tilde{\beta})$, which is explicitly given by the following equation,

$$\cos(2\tilde{\beta}) = \frac{v_1'^2 + v_2'^2 - v_1^2 - v_2^2}{v^2}. \tag{5.51}$$

For this reason, the parameter space $\cos(2\tilde{\beta})$ vs g_X was also plotted. This is shown in the figure (5.3).

Negative values of $\cos(2\tilde{\beta})$ are found in the Montecarlo simulation because the primed VEVs happened to be less than non-primed ones. This comes in part from the hierarchy between the top quark, whose mass is proportional to v_1, and the bottom quark, whose mass is proportional to v_2'. Thus, $v_1 > v_2'$. The other reason can be seen in equation (5.48), and it is that the second term, that we called Δm_h^2, is proportional to difference between squared primed and non-primed VEVs. Since that contribution has to be positive for reproducing the 125 GeV Higgs boson, then $v_1, v_2 > v_1', v_2'$. The second term Δm_h^2 is also proportional to g_X^2. Thereby, the greater the coupling constant g_X, the smaller the differences between squared primed and non-primed VEVs are needed. These constraints on the $U(1)_X$ will be important when analysing the phenomenology of the Z' boson.

5.2.4 Gauge boson masses

The masses of the gauge bosons are explained by the kinetic terms of the scalar particles. The covariant derivative adds one term compared to the SM, because of the new gauge symmetry $U(1)_X$. It is given by:

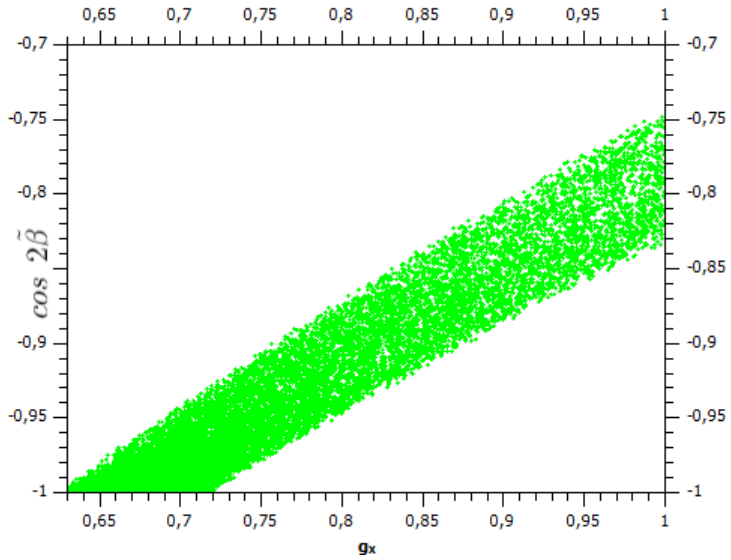

Figure 5.3: Region in the parameter space $\cos 2\tilde{\beta}$ vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

$$D_\mu = \partial_\mu - igW_\mu^a T_a - ig'\frac{Y}{2}B_\mu - ig_X B'_\mu. \tag{5.52}$$

After the spontaneous symmetry breaking, mass terms for the gauge boson were found. The mass of the charged bosons $W_\mu^\pm = \frac{W_\mu^1 \pm W_\mu^2}{\sqrt{2}}$ is the same as the one obtained in the SM. Thus, in this model:

$$M_W = \frac{gv}{2}. \tag{5.53}$$

The mass terms of neutral gauge bosons can be arranged in a matrix form like $V_G^\dagger M_0^2 V_G$, where $V_G = (W_\mu^3, B_\mu, B'_\mu)$ is the basis. The 3×3 mass matrix for neutral gauge bosons M_0^2 is shown right ahead,

$$M_0^2 = \frac{1}{4}\begin{pmatrix} g^2v^2 & -gg'v^2 & -\frac{2}{3}gg_Xv^2(1+\cos^2\beta) \\ * & g'^2v^2 & \frac{2}{3}g'g_Xv^2(1+\cos^2\beta) \\ * & * & \frac{4}{9}g_X^2\left[V_\chi^2 + (1+3\cos^2\beta)v^2\right] \end{pmatrix}. \tag{5.54}$$

In the matrix, for making expressions shorter, the following terms have been defined:

$$\tan\beta = \frac{\sqrt{v_2^2 + v_2'^2}}{\sqrt{v_1^2 + v_1'^2}} \equiv \frac{V_2}{V_1} \tag{5.55}$$

$$V_\chi^2 \equiv v_\chi^2 + v_\chi'^2 \tag{5.56}$$

The rank of M_0^2 turns out to be 2, so there is one massless gauge boson, which should be of course the photon. Solving the characteristic equation, the mass eigenvalues of the

neutral interaction bosons are:

$$M_\gamma = 0, \tag{5.57}$$

$$M_Z \approx \frac{gv}{2\cos\theta_W}, \tag{5.58}$$

$$M_{Z'} \approx \frac{g_X V_\chi}{3}, \tag{5.59}$$

The mass of the Z boson is approximately the same as the one predicted in the SM, recalling that $\tan\theta_W = \frac{g'}{g}$. It actually gets corrections of the form $\sim 1/V_\chi$, but those are suppressed. The mass of the new interaction boson Z' lies naturally much higher than the electroweak since it is proportional to V_χ. Mass eigenstates and the basis V_G are related by an unitary rotation that diagonalizes M_0^2. This relation is explicitly shown in the next equation,

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W & 0 \\ \cos\theta_W\cos\theta_Z & -\sin\theta_W\cos\theta_Z & \sin\theta_Z \\ -\cos\theta_W\sin\theta_Z & \sin\theta_W\sin\theta_Z & \cos\theta_Z \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \\ B'_\mu \end{pmatrix}, \tag{5.60}$$

where a new angle parameter θ_Z has been defined as:

$$\sin\theta_Z = (1+\cos^2\beta)\frac{2g_X\cos\theta_W}{3g}\left(\frac{M_Z}{M_{Z'}}\right)^2. \tag{5.61}$$

Due to the fact that a Z' has not been observed, the relation $M_{Z'} \gg M_Z$ must be true. Then, θ_Z is small. In the limit of ininitely small angle θ_Z, the SM Z boson plus a totally decoupled Z' boson is obtained. However, this is not the case, and the interactions between neutral bosons change compared to the SM. This should be constrained in the light of the Large Hadron Collider data.

Chapter 6

Conclusions

The Higgs mass is a puzzling feature of today's particle physics. Seen from the Standard Model, its mass is perfectly explained. However, taking a step beyond this model with the known tools, a loss of predictabillity is origined, due to a sensibility between widely separated energy scales. Thus, the Standard Model fails at being natural as an effective field theory. The supersymmetry prevents this from happening, and its most direct version of the Standard Model is the Minimal Supersymmetric Standard Model. Although there is a gain of naturalness in the Higgs mass, many new issues have to be solved or even forgotten. Between them, the Higgs mass happens to fall short at the leading order, having as a goal the measured 125 GeV boson. Thereby, radiative corrections at the order of the tree level must be assumed in the Minimal Supersymmetric Standard Model. This supersymmetric version can push the regions of its parameters to get the 130 GeV as an upper bound to the Higgs mass. However, this has been considered unnatural.

Different alternatives to the Minimal Supersymmetric Standard Model have been proposed, being the model presented here one of them. It came from a minimal supersymmetrization of a model that follows the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ [64], with $U(1)_X$ non-universal due to the charge assignation. Thus, to cancel chiral anomalies, the model consists on four Higgs doublets $\phi_1, \phi_1', \phi_2, \phi_2'$ and four Higgs singlets $\chi, \chi', \sigma, \sigma'$. The variety of doublets forms a suitable scenario for the fermion mass hierarchy, giving to the vacuum expectation values a preferred parameter space. Since the mass of the top quark turns out to be practically proportional to v_1, then, $170\text{GeV} < v_1 < 200\text{GeV}$. In a similar way, the mass of the bottom quark is proportional to v_2', and therefore, $3\text{GeV} < v_2' < 7\text{GeV}$. The singlets are necessary for giving exotic fermions a free parameter mass and to break the $U(1)_X$. The scalar spectrum of the Higgs sector was found. Three different groups of scalar states could be analysed separately: CP-even, CP-odd and charged. The masses of the particles were obtained by assuming a seesaw mechanism between the different mass parameters that were in the model.

The CP-even particles are the eight massive states $h_1, h_2, h_3, h_4, h_5, h_6, h_7$ and h_8, listed from the lightest to the heaviest. The heaviest particles h_5, h_6, h_7 and h_8 have a mass depending mainly on the parameters from the interaction with singlets in the full potential, which are μ_σ, $m_\sigma, m_\sigma', v_\chi$ and v_χ'. In an intermediate energy scale, but heavier than the ones reached so far (14 TeV), there are the particles h_2, h_3 and h_4. The masses of these states rely mostly on the mass parameters $\mu_{11}, \mu_{22}, k_i v_j (v_j')$, with $i = 1, 2, 3, 4$ and $j = 1, 2$. The lightest CP-even particle, h_1, has a mass naturally at the electroweak scale, because it depends on the electroweak parameters as the way to establish the mass scale. Moreover, it depends on the $U(1)_X$ coupling constant g_X, that comes from the D-terms. In this way, the Higgs mass can reach the 125 GeV at tree level, without the need of large

radiative corrections. Then, a radom search with the Montecarlo method was used to explore the space parameter of the electroweak vacuum expectation values v_1, v_2, v_1' and v_2', and the coupling constant g_X. The condition for accepting a value was to be in agreement with a Higgs boson of (125.3 ± 0.4) GeV at 95% confidence level. v_1 and v_2' were searched in the intervals imposed by the fermion mass hierarchy and the other ones were constrained to the condition (5.50), to obtain the right masses for the W and Z bosons. $0 < g_X < 1$ was the exploration region of the $U(1)_X$ coupling constant. The results of the search can be seen in figures (5.1), (5.2) and (5.3). The exploration resulted in $g_X > 0.63$, $0\text{GeV} < v_1' < 80\text{GeV}$ and $112\text{GeV} < v_2 < 180\text{GeV}$. This happened because the contribution from the D-terms to the Higgs mass depends on differences between non-primed and primed vacuum expectaction values. Thereby, $v_1, v_2 > v_1', v_2'$.

The charged spectrum consists in a massless goldstone boson $G_W^{\pm}$, that gives mass to the charged W boson, and three massive states H_1, H_2 and H_3. The masses of these last particles depend for the most part in heavy mass parameters from the soft-supersymmetry breaking potential and couplings with the singlet fields. The CP-odd particles are conformed of two massless states η_1 and η_5, which are the would-be golstone bosons that give mass to the Z and Z' interaction bosons. All other states, $\eta_2, \eta_3, \eta_4, \eta_6, \eta_7$ and η_8 have masses relying largely on the mass parameters different from the electroweak ones. Therefore, they all are naturally heavy. The interaction boson masses are consistent with the results in the Standard Model, but there is a mixing between the Z and the Z' parameterized with an angle parameter θ_Z, which is given in equation (5.61).

Although some features of the model have been explored, many others remained without analysis. For instance, flavor changing neutral currents exist here, and they have to be constrained with the upper limits on this behaviour. The μ problem, which is usually solved in extensions of the Minimal Supersymmetric Standard Model, is also an important field to study in this model. Finally to mention here, it was found in the parameter region the possibility of an inert Higgs, which may serve as a dark matter candidate. The list is surely larger than the few aspects named here, but the first steps in the exploration of the supersymmetric model have been done.

Appendix A

Seesaw rotation of $n \times n$ matrices

The mass matrices, that appeared for the scalar fields in the Higgs sector of the model, had to be diagonalized using a seesaw rotation. Using a base $V = (\phi_1, \phi_2, ..., \phi_n)$, where the $\{\phi\}_{i=1,2,...,n}$ are scalar fields, a mass matrix whose eigenvalues are the quadratic masses can be expressed as:

$$\mathcal{M}_V = \begin{pmatrix} \mathcal{M}^l_{m\times m} & \mathcal{M}^{lh}_{m\times n-m} \\ (\mathcal{M}^{lh}_{m\times n-m})^T & \mathcal{M}^h_{n-m\times n-m} \end{pmatrix}. \tag{A.1}$$

The seesaw mechanism can be withdrawn from the scheme used in massive neutrinos to any mass matrix that has a scale hierarchy. That is why the superscripts l for light and h heavy are used. Assuming the following scale hierarchy amongst the entries of the matrix blocks,

$$\mathcal{O}(\mathcal{M}^h) > \mathcal{O}(\mathcal{M}^{lh}), \mathcal{O}(\mathcal{M}^l), \tag{A.2}$$

a seesaw rotation was applied to separate the heavy part from the light part [90]. The seesaw rotation is done with the following matrix,

$$\mathbb{U}^V_{SS} = \begin{pmatrix} 1 & \Theta^{V\dagger} \\ -\Theta^V & 1, \end{pmatrix} \tag{A.3}$$

with $\Theta^V = (\mathcal{M}^l)^{-1}\mathcal{M}^{lh}$. This leaves the matrix in a block-diagonal form, as it was required

$$(\mathbb{U}^V_{\rm SS})^T \mathcal{M}_V \mathbb{U}^V_{\rm SS} = \begin{pmatrix} m^l_V & 0_{m\times n-m} \\ 0_{n-m\times m} & M^h_V \end{pmatrix}, \tag{A.4}$$

where $m^l_V \approx \mathcal{M}^l - \mathcal{M}^{lh}(\mathcal{M}^h)^{-1}(\mathcal{M}^{lf})^T$ is the mass matrix for the lighter states and $M^h_V \approx \mathcal{M}^h$ the one for the heavier states.

Appendix B

Solutions of quartic equations

In most of the mass matrices that were diagonalized, it was found that a solution to a quartic equation was required. Here the solutions to a general quartic equation are shown. Let us have the following quartic equation:

$$ax^4 + bx^3 + cx^2 + dx + e = 0, \tag{B.1}$$

with $a \neq 0$.

The Ferrari method consists in reducing the quartic equation (B.1) into two quadratic and one cubic equations [91], whose solutions are known. Therefore, the solutions of the quartic equation (B.1) turn out to be:

$$x = \frac{-p \pm \sqrt{p^2 - 8q}}{4}, \tag{B.2}$$

where p and q are given by:

$$p = \frac{b}{a} \pm \sqrt{\frac{b^2}{a^2} - \frac{4c}{a} + 4y}, \qquad q = y \mp \sqrt{y^2 - \frac{4e}{a}}. \tag{B.3}$$

The y comes from the solution of a cubic equation given by:

$$y^3 - \frac{c}{a}y^2 + \left(\frac{bd}{a^2} - \frac{4e}{a}\right)y + \left(\frac{4ce}{a^2} - \frac{b^2e}{a^3} - \frac{d^2}{a^2}\right). \tag{B.4}$$

The solution of this cubic equation can be found by Cadano's formula, which states the general solutions. Let $B = -\frac{b}{a}$, $C = \left(\frac{bd}{a^2} - \frac{4e}{a}\right)$ and $D = \left(\frac{4ce}{a^2} - \frac{b^2e}{a^3} - \frac{d^2}{a^2}\right)$, the general solutions of the cubic equation is shown right ahead,

$$\begin{aligned} y_1 &= S + T - \frac{B}{3}, \\ y_2 &= -\frac{S+T}{2} - \frac{B}{3} + \frac{i\sqrt{3}}{2}(S - T), \\ y_3 &== -\frac{S+T}{2} - \frac{B}{3} - \frac{i\sqrt{3}}{2}(S - T). \end{aligned} \tag{B.5}$$

The expressions of S and T are the following,

$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}}, \qquad T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}, \tag{B.6}$$

with:

$$Q = \frac{3C - B^2}{9}, \qquad R = \frac{9BC - 27D - 2B^3}{54}. \tag{B.7}$$

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