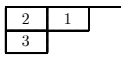
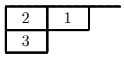
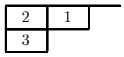
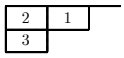
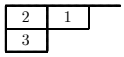
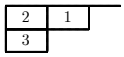
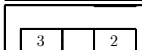
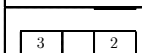
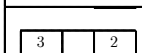
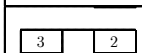
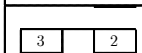
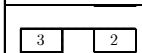
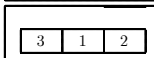
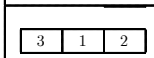
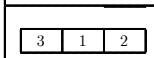
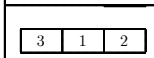
2 Puentes:

Cruces	Invariante Λ	Pol. Swallowek	Jones	C. Euler
	$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}^*$	$-\frac{x^2}{y} - x^2 - yx + \frac{x}{y} + y + 1$	pol_5	0
	$\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & -2 & 0 \end{pmatrix}$	$yx^2 - \frac{x^2}{y} + y^2x - yx + \frac{x}{y} - x - y^2 + 1$	1	-2
	$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}$	$\frac{x^3}{y} + x^3 - \frac{x^2}{y} - x^2 - y^2x - yx + y^2 + y$	1	-2
	$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$	$\frac{x^3}{y} + \frac{x^3}{y^2} - \frac{x^2}{y} - \frac{1}{x^2y^2} - yx - x + y + 1$	1	-2
	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$	$-\frac{x^2}{y^2} + x^2 + yx - \frac{x}{y} + \frac{x}{y^2} - x - y + \frac{1}{y}$	1	-2
	$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^*$	$\frac{x^2}{y} + x^2 + yx - \frac{x}{y} - y - 1$	pol_6	0

Cruces	Invariante Λ	Pol. Swallowek	Jones	C. Euler
	$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^*$	$-\frac{x^2}{y} - x^2 - yx + \frac{x}{y} + y + 1$	pol_5	0
	$\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$	$-\frac{x^3}{y} - x^3 + \frac{x^2}{y} + x^2 + y^2x + yx - y^2 - y$	1	-2
	$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$	$-yx^2 + \frac{x^2}{y} - y^2x + yx - \frac{x}{y} + x + y^2 - 1$	1	-2
	$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}$	$\frac{x^2}{y^2} - x^2 - yx + \frac{x}{y} - \frac{x}{y^2} + x + y - \frac{1}{y}$	1	-2
	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & -2 & 0 \end{pmatrix}$	$-\frac{x^3}{y} - \frac{x^3}{y^2} + \frac{x^2}{y} + \frac{1}{x^2y^2} + yx + x - y - 1$	1	-2
	$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}^*$	$\frac{x^2}{y} + x^2 + yx - \frac{x}{y} - y - 1$	pol_6	0

3 Puentes o Alternantes

Cruces	Invariante Λ	Pol. Swallowek	Jones	C. Euler
	$\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^*$	0	pol_3	2
	$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & -2 & -1 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$	$\frac{x^3}{y^2} - x^3 - \frac{x^2}{y^2} + x^2 + y^2x - x - y^2 + 1$	1	0
	$\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -2 \\ 1 & 2 & 0 \end{pmatrix}$	$-\frac{x^3}{y^2} + x^3 + \frac{x^2}{y^2} - x^2 - y^2x + x + y^2 - 1$	1	0
	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^*$	0	pol_4	2

Tenemos así que hay 10 nudos de 1 puente, 12 nudos de 2 puentes y 4 nudos de tres puentes o alternantes, para un total de 26 nudos. Entre ellos hay 2 clásicos, que son el trébol derecho e izquierdo. El invariante Λ los agrupa en 24 grupos, es decir, no puede distinguir los dos tréboles y otra pareja de nudos. El polinomio Z los agrupa en 21 grupos, y distingue los dos nudos virtuales que no distingue el invariante Λ. El polinomio de Jones los agrupa en sólo 5 grupos, pero logra distinguir los dos tréboles.

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