Proyecto de Investigación:

Modelos de Unificación de Interacciones Fundamentales basados en el Grupo de Simetría $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$[1]

INFORME FINAL

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Sede Medellín

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Informe final de investigación

1. Ficha del proyecto

1. **Título:** "Modelos de Unificación de Interacciones Fundamentales basados en el Grupo de Simetría $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$".

2. **Código DIME:** 030802739.

3. **Facultad:** Ciencias.

4. **Investigador Principal:** Luis Alberto Sánchez Duque. Intesidad horaria: 10 horas/semana.

5. **Monitor Académico de investigación:** Felipe Andrés Pérez Valencia, Carnet 199906863, estudiante de Ingeniería Física.

6. **Fecha de inicio:** Enero de 2004.

7. **Fecha de terminación:** Enero de 2005.

2. Descripción de proyecto

2.1. Resumen ejecutivo de la idea central

En el presente proyecto se propone la identificación y construcción de modelos de unificación de interacciones fuertes y electro-débiles usando el grupo de simetría gauge $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$, en los cuales: (a) no existan partículas con cargas eléctricas exóticas ni en el sector de bosones de gauge ni en el sector fermiónico, con el objeto de simplificar la fenomenología, y (b) se satisfaga la condición de libertad de anomalías quirales, con el objeto de que los modelos sean renormalizables (que los efectos cuánticos sean calculables). Se pretende además estudiar las implicaciones fenomenológicas de por lo menos uno de los modelos identificados con el objeto de contrastarlo con los datos experimentales para determinar cuál es su viabilidad como modelo realista.

2.2. Objetivos del proyecto

2.2.1. Objetivo general

Identificación y construcción de modelos realistas de unificación de interacciones fuertes y electro-débiles, basados en el grupo de simetría gauge $SU(3)_c \otimes SU(4)_L \otimes U(1)_x$, libres de anomalías quirales y sin cargas eléctricas exóticas.

2.2.2. Objetivos específicos

1. Estudio del generador de carga eléctrica en la simetría $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ para determinar unívocamente las condiciones bajo las cuales no se presentan cargas eléctricas exóticas.

2. Estudio de la estructura de multipletes y de la asignación de números cuánticos que dan lugar a modelos libres de anomalías quirales.

3. Estudio del mecanismo de rompimiento de simetría y de generación de masas en por lo menos uno de los modelos para identificar posibles indicios de jerarquías.

4. Estudio de las corrientes cargadas y neutras que permita confrontar por lo menos uno de los modelos con las cotas experimentales provenientes de la medición del ancho de decaimiento del bosn Z^0 y de la medición de violación de paridad atómica en Cesio.

3. Resultados directos

3.1. Aportes al conocimiento científico

Por primera vez en la literatura científica se hizo un estudio sistemático de todos los posibles modelos sin cargas eléctricas exóticas que pueden construirse con base en la simetría $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ y se identificaron seis nuevos modelos, antes no conocidos, cuatro de los cuales son modelos en los que las anomalías quirales se cancelan entre las diferentes familias de partículas elementales (modelos de tres familias), mientras que en dos de ellos esas anomalías se cancelan familia por familia (modelos de una familia).

Adicionalmente logró mostrarse que dos de los modelos de tres familias son modelos realistas en dos sentidos: (a) proporcionan un espectro de masas para los fermiones fundamentales conocidos el cual es consistente con los datos experimentales, y (b) los cálculos de la masa del nuevo bosón de gauge neutro $Z^{0\prime}$ que predicen estos modelos y de su ángulo de mezcla son consistentes con las cotas experimentales provenientes de las mediciones del ancho de decaimiento del bosón neutro Z^0 y de la violación de paridad atómica en Cesio.

3.2. Implementación de Software para investigación

Se implementó un software en lenguage C el cual se usó para la confrontación de las predicciones de los modelos estudiados con los resultados experimentales, en particular, con los datos experimentales sobre la masa y el ángulo de mezcla del nuevo bosón de gauge Z'^0 cuya existencia es predicha por estos modelos.

El programa, el cual se presenta en el anexo 3, llama a una subrutina (optimization.C) en la cual se usa información pertinente de cada modelo y, con el uso de librerías del CERN (Centro Europeo para la Investigación Nuclear), se minimiza la función χ^2 para obtener la región de confianza para la masa y el ángulo de mezcla mencionados.

Este software será útil en un futuro para confrontar experimentalmente las predicciones de otros modelos en física de partículas elementales.

4. Resumen ejecutivo de los resultados indirectos

4.1. Formación de recursos humanos

- Se culminó del trabajo de Grado "Conexiones entre las simetrías de la Relatividad Especial, la Física de Partículas Elementales y la Optica" del estudiante de Ingeniería Física y monitor académico de investigación del proyecto Felipe Andrés Pérez Valencia, bajo la dirección del investigador principal del presente proyecto. Este trabajo fue presentado públicamente el 31 de Enero de 2005 y el estudiante recibió su grado de Ingeniero Físico el pasado 30 de Marzo.

- El estudiante Pérez Valencia participó activamente en el desarrollo del proyecto con la implementación del software que se describió en la sección anterior y aparece como uno de los co-autores en una de las publicaciones que se listan más adelante.

4.2. Publicaciones

Los resultados de la presente investigación han sido publicados en dos revistas internacionales indexadas y de tipo A en la clasificación de Colciencias. Esas publicaciones son:

- William A. Ponce, Diego A. Gutiérrez y Luis A. Sánchez: "$SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ without exotic electric charges", Physical Review **D69**, 055005 (2004).

- Luis A. Sánchez, Felipe A. Pérez y William A. Ponce: "$SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ model for three families", The European Physical Journal C, **35**, 259 (2004).

4.3. Cooperación con otros grupos de investigación

El presente proyecto de investigación fue útil en el fortalecimiento de la cooperación con el Grupo de Fenomenología de las Interacciones Fundamentales del Instituto de Física de la Universidad de Antioquia; en efecto, dos de los co-autores de las publicaciones: William Ponce y Diego Gutiérrez, son integrantes de ese grupo de investigación.

DIFICULTADES

Las mayores dificultad se encontraron tanto en el retraso con el que llegaron los dineros asignados al proyecto como en la ejecución de algunos rubros, lo cual, en ocasiones, se traduce en retraso en la ejecución misma del proyecto.

5. Anexos

Se presentan a continuación los siguientes anexos:

- **Anexo 1:** Resumen del Trabajo de Grado del Estudiante Felipe Andrés Pérez Valencia.
- **Anexo 2:** Publicaciones.
- **Anexo 3:** Software.

Anexo 1

TRABAJO DE GRADO

Título: Conexiones entre las simetrías de la Relatividad Especial, la Física de Partículas Elementales y la Optica.

Presentado por: Felipe Andrés Pérez Valencia.
(como requisito parcial para optar al título de Ingeniero Físico.)

Director: Luis Alberto Sánchez Duque.

Fecha de Sustentación: Enero 31 de 2005.

Jurado: Profesores William A. Ponce Gutiérrez, Ph.D., Instituto de Física de la Universidad de Antioquia, y Javier Morales Aramburo, M.Sc., Escuela de Física Universidad Nacional - sede Medellín.

Concepto del Jurado: Aprobado con recomendacón de la distinción "Meritoria".

Resumen:

Se hace una descripción detallada de los grupo de Lorentz y del grupo $SL(2,C)$, y se muestra el homomorfismo existente entre ambos grupos y su conexión con los "little groups" de Wigner, que son los subgrupos máximos del grupo de Lorentz que dejan invariante el cuadrimomentum p^{μ} de una partícula. Se estudia además la relación que existe entre los generadores de traslaciones y las transformaciones gauge para partículas no masivas, y se muestra la relación entre el generador de rotaciones y la helicidad de esas partículas en el contexto de la mecánica cuántica relativista. Posteriormente se muestra el procedimiento para obtener el grupo $E(2)$ de transformaciones en el plano euclidiano a partir del grupo $O(3)$ de rotaciones en tres dimensiones, llamado contracción de Inonu-Wigner, y se utiliza este mismo procedimiento para contraer $O(3)$ como little group a $E(2)$ como little group. Finalmente se estudia un ejemplo concreto de cómo un sistema de una lente inmersa en aire puede describirse a través de una matriz que se puede escribir mediante dos transformaciones de similaridad por medio de transformaciones de Lorentz, una para partículas masivas y otra para partículas no masivas.

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En las dos páginas siguientes se presentan fotocopias de la portada y del índice del trabajo de grado, así como del acta de sustentación del mismo.

CONEXIONES ENTRE LAS SIMETRIAS DE LA RELATIVIDAD ESPECIAL, LA FISICA DE PARTICULAS ELEMENTALES Y LA OPTICA.

Trabajo Dirigido de Grado

presentado por

Felipe Andrés Pérez Valencia

a la

Facultad de Ciencias

como requisito parcial

para optar al título

Ingeniero Físico

Trabajo Dirigido por

Luis Alberto Sánchez Duque

Universidad Nacional de Colombia

Sede Medellín

Octubre 2004

Contenido

UNIVERSIDAD NACIONAL DE COLOMBIA

Sede Medellín

REGISTRO DE CALIFICACIONES ESPECIALES

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ESCUELA DE: FÍSICA
PROFESOR: LUIS ALBERTO SÁNCHEZ DUQUE
ASIGNATURA: TRABAJO DIRIGIDO II

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	Titulo *"CONEXIONES ENTRE LAS SIMETRÍAS DE LA RELATIVIDAD ESPECIAL, LA FÍSICA DE PARTÍCULAS ELEMENTALES Y LA ÓPTICA"*		
	JAVIER MORALES ARAMBURO Escuela de Fisica Universidad Nacional Sede Medellín Jurado		Aprobado
	WILLIAM PONCE Instituto de Física Universidad de Antioquia Jurado		Aprobado

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Beatriz Elena Correa Restrepo
Secretaria Academica

OBSERVACIONES: SE SUSTENTÓ EL DÍA LUNES 31 DE ENERO DE 2005, A LAS 10:00 HORAS EN EL AULA 21-420

Recomendamos mención "Meritoria"

FIRMA DEL PROFESOR

VoBo Claudia García
COORDINADOR PROGRAMAS CURRICULARES
ESCUELA DE FISICA

Anexo 2

PUBLICACIONES

$SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ without exotic electric charges

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(Received 10 December 2003; published 31 March 2004)

We present an extension of the standard model to the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ with a family nonuniversal treatment and anomalies canceled among the three families in a nontrivial fashion. The mass scales, the gauge boson masses, and the masses for the spin 1/2 particles in the model are analyzed. The neutral currents coupled to all neutral vector bosons in the model are studied, and particular values of the parameters are used in order to simplify the mixing between the three neutral currents present in the theory, mixing which is further constrained by experimental results from the CERN LEP, SLAC Linear Collider, and atomic parity violation.

DOI: 10.1103/PhysRevD.69.055007 PACS number(s): 12.10.Dm, 12.15.Ff, 12.60.Cn

I. INTRODUCTION

In spite of the overwhelming phenomenological success of the standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, with $SU(2)_L \otimes U(1)_Y$ hidden and $SU(3)_c$ confined [1], it fails to explain several issues such as hierarchical fermion masses and mixing angles, charge quantization, strong *CP* violation, replication of families and neutrino oscillations among others. For example, in the weak basis, before symmetry is broken, the three families in the SM are identical to each other; when symmetry breaking takes place, the fermions get masses according to their experimental values and the three families acquire a strong hierarchy. However in the SM there is no mechanism for explaining the origin of families or the fermion mass spectrum.

These drawbacks of the SM have led to a strong belief that the model is still incomplete and that it must be regarded as a low-energy effective field theory originating from a more fundamental one. That belief lies on strong conceptual indications for physics beyond the SM which have produced a variety of theoretically well motivated extensions of the model: left-right symmetry, grand unification, supersymmetry, superstring inspired extensions, etc. [2].

At present the only experimental fact that points toward a beyond the SM structure lies in the neutrino sector, and even there the results are not final yet. So a reasonable approach is to depart from the SM as little as possible, allowing some room for neutrino oscillations [3].

$SU(4)_L \otimes U(1)_X$ as a flavor group has been considered before in the literature [4,5], and, among its best features, provides an alternative to the problem of the number N_f of families, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, N_c being the number of colors of $SU(3)_c$ (also known as QCD). In addition, this gauge structure has been used recently in order to implement the so-called little Higgs mechanism [5].

In this paper an analysis of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ local gauge theory (hereafter the 3-4-1 theory) shows that, by restricting the fermion field representations to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are obtained, while by relaxing the condition of the nonexistence of exotic electric charges, an infinite number of models can be generated.

This paper is organized as follows. In the next section we introduce the model based on the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ which we are going to study. In Sec. III we describe the scalar sector needed to break the symmetry and to produce masses to the fermion fields in the model. In Sec. IV we study the gauge boson sector paying special attention to the neutral currents present in the model and their mixing. In Sec. V we analyze the fermion mass spectrum. In Sec. VI we use experimental results in order to constrain the mixing angle between two of the neutral currents and the mass scale of the new neutral gauge bosons. In the last section we summarize the model and state our conclusions. At the end an Appendix is presented in which we make a systematic analysis of the 3-4-1 symmetry and obtain general conditions to have anomaly free models without exotic electric charges.

II. THE FERMION CONTENT OF THE MODEL

In what follows we assume that the electroweak gauge group is $SU(4)_L \otimes U(1)_X$ which contains $SU(2)_L \otimes U(1)_Y$ as a subgroup, with a nonuniversal hypercharge X in the quark sector, which in turn implies anomaly cancellation among the families in a nontrivial fashion. We also assume that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform either under the 4 or $\bar{4}$ fundamental representations of $SU(4)_L$, and that as in the SM, $SU(3)_c$ is vectorlike.

With the former assumptions we look for the simplest structure in such a way that, not only it does not contain fields with exotic electric charges, but also that charged exotic leptons are absent from the anomaly-free spectrum. According to the Appendix there is only one model (model A) satisfying all those constraints, for which the electric charge operator is given by $Q=T_{3L}+(1/\sqrt{3})T_{8L}+(1/\sqrt{6})T_{15L}+XI_4$, with the following fermion structure:

$$Q_{aL}=\begin{pmatrix} u_a \\ d_a \\ D_a \\ D'_a \end{pmatrix}_L \quad u^c_{aL} \quad d^c_{aL} \quad D^c_{aL} \quad D'^c_{aL}$$
$$[3,4,-\tfrac{1}{12}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,\tfrac{1}{3}]$$

$$Q_{1L}=\begin{pmatrix} d_1 \\ u_1 \\ U_1 \\ U'_1 \end{pmatrix}_L \quad d^c_{1L} \quad u^c_{1L} \quad U^c_{1L} \quad U'^c_{1L}$$
$$[3,\bar{4},\tfrac{5}{12}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,-\tfrac{2}{3}]$$

$$L_{\alpha L}=\begin{pmatrix} e^-_\alpha \\ \nu_{e\alpha} \\ N^0_\alpha \\ N'^0_\alpha \end{pmatrix}_L \quad e^+_\alpha$$
$$[1,\bar{4},-\tfrac{1}{4}] \quad [1,1,1]$$

where $a=2,3$ and $\alpha=1,2,3$ are two and three family indexes, respectively. The numbers in parentheses refer to the $[SU(3)_C,SU(4)_L,U(1)_X]$ quantum numbers, respectively. Notice that if needed, the lepton structure of the model can be augmented with an undetermined number of neutral Weyl singlet states $N^0_{L,b}\sim[1,1,0]$, $b=1,2,\ldots$, without violating our assumptions, neither the anomaly constraint relations, because singlets with no X charges are as good as not being present as far as anomaly cancellation is concerned.

III. THE SCALAR SECTOR

Our aim is to break the symmetry following the pattern

$$SU(3)_c\otimes SU(4)_L\otimes U(1)_X \rightarrow SU(3)_c\otimes SU(3)_L\otimes U(1)_X$$
$$\rightarrow SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$$
$$\rightarrow SU(3)_c\otimes U(1)_Q,$$

where $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ refers to the so-called 3-3-1 structure introduced in Ref. [6]. At the same time we want to give masses to the fermion fields in the model. With this in mind we introduce the following three Higgs scalars: $\phi_1[1,4,-3/4]$ with a vacuum expectation value (VEV) aligned in the direction $\langle\phi_1\rangle=(v,0,0,0)^T$; $\phi_2[1,\bar{4},-1/4]$ with a VEV aligned as $\langle\phi_2\rangle=(0,0,V,0)^T$ and $\phi_3[1,\bar{4},-1/4]$ with a VEV aligned as $\langle\phi_3\rangle=(0,0,0,V')^T$, with the hierarchy $V\sim V'\gg v\sim 174$ GeV (the electroweak breaking scale).

IV. THE GAUGE BOSON SECTOR

In the model there are a total of 24 gauge bosons: One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 15 gauge fields associated with $SU(4)_L$ which we may write as

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha=\frac{1}{\sqrt{2}}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} & X^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} & X^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 & Y^{0\mu} \\ X^{-\mu} & \bar{X}^{0\mu} & \bar{Y}^{0\mu} & D^\mu_4 \end{pmatrix},$$

where $D^\mu_1=A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, $D^\mu_2=-A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$; $D^\mu_3=-2A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, and $D^\mu_4=-3A^\mu_{15}/\sqrt{12}$.

After breaking the symmetry with $\langle\phi_1\rangle+\langle\phi_2\rangle+\langle\phi_3\rangle$ and using for the covariant derivative for 4-plets $iD^\mu=i\partial^\mu-(g/2)\lambda_\alpha A^\mu_\alpha-g'XB^\mu$, where g and g' are the $SU(4)_L$ and $U(1)_X$ gauge coupling constants, respectively, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm}=(g^2/2)v^2$ as in the SM, $M^2_{K^\pm}=(g^2/2)(v^2+V^2)$, $M^2_{X^\pm}=(g^2/2)(v^2+V'^2)$, $M^2_{K^0(\bar{K}^0)}=(g^2/2)V^2$, $M^2_{X^0(\bar{X}^0)}=(g^2/2)V'^2$ and $M^2_{Y^0(\bar{Y}^0)}=(g^2/2)(V^2+V'^2)$. Since $W^\pm$ does not mix with $K^\pm$ or with $X^\pm$ we have that $v\approx 174$ GeV as in the SM.

For the four neutral gauge bosons we get mass terms of the form

$$M=\frac{g^2}{2}\left[V^2\left(\frac{g'B^\mu}{2g}-\frac{2A^\mu_8}{\sqrt{3}}+\frac{A^\mu_{15}}{\sqrt{6}}\right)^2+V'^2\left(\frac{g'B^\mu}{2g}-\frac{3A^\mu_{15}}{\sqrt{6}}\right)^2+v^2\left(A^\mu_3+\frac{A^\mu_8}{\sqrt{3}}+\frac{A^\mu_{15}}{\sqrt{6}}-\frac{3g'B^\mu}{2g}\right)^2\right].$$

M is a 4×4 matrix with a zero eigenvalue corresponding to the photon. Once the photon field has been identified, we remain with a 3×3 mass matrix for three neutral gauge bosons Z^μ, Z'^μ and Z''^μ. Since we are interested now in the low energy phenomenology of our model, we can choose $V=V'$ in order to simplify matters. For this particular case the field $Z''^\mu=A^\mu_8/\sqrt{3}-\sqrt{2/3}A^\mu_{15}$ decouples from the other two and acquires a squared mass $(g^2/2)V^2$. By diagonalizing the remaining 2×2 mass matrix we get the other two physical neutral gauge bosons which are defined through the mixing angle θ between Z_μ, Z'_μ:

$$Z^\mu_1=Z_\mu\cos\theta+Z'_\mu\sin\theta,$$

$$Z^\mu_2=-Z_\mu\sin\theta+Z'_\mu\cos\theta,$$

where

$$\tan(2\theta)=-\frac{2\sqrt{2}C_W}{\sqrt{1+2\delta^2}\left[1+\frac{2V^2}{v^2}C_W^4-\frac{2}{1+2\delta^2}C_W^2\right]}, \quad (1)$$

with $\delta=g'/(2g)$.

The photon field A^μ and the fields Z_μ and Z'_μ are given by

$$A^\mu=S_W A_3^\mu+C_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu+\frac{A_{15}^\mu}{\sqrt{2}}\right)+(1-T_W^2/2)^{1/2}B^\mu\right],$$

$$Z^\mu=C_W A_3^\mu-S_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu+\frac{A_{15}^\mu}{\sqrt{2}}\right)+(1-T_W^2/2)^{1/2}B^\mu\right],$$

$$Z'^\mu=\sqrt{\frac{2}{3}}(1-T_W^2/2)^{1/2}\left(A_8^\mu+\frac{A_{15}^\mu}{\sqrt{2}}\right)-\frac{T_W}{\sqrt{2}}B^\mu. \quad (2)$$

$S_W=2\delta/\sqrt{6\delta^2+1}$ and C_W are the sine and cosine of the electroweak mixing angle respectively, and $T_W=S_W/C_W$. We can also identify the Y hypercharge associated with the SM Abelian gauge boson as

$$Y^\mu=\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu+\frac{A_{15}^\mu}{\sqrt{2}}\right)+(1-T_W^2/2)^{1/2}B^\mu\right]. \quad (3)$$

A. Charged currents

After some algebra, the Hamiltonian for the charged currents can be written as

$$\begin{aligned}H^{CC}=\frac{g}{\sqrt{2}}\Bigg[&W_\mu^+\left(\left(\sum_{a=2}^{3}\bar{u}_{aL}\gamma^\mu d_{aL}\right)-\bar{u}_{1L}\gamma^\mu d_{1L}-\left(\sum_{\alpha=1}^{3}\bar{\nu}_{e\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right)\\
&+K_\mu^+\left(\left(\sum_{a=2}^{3}\bar{u}_{aL}\gamma^\mu D_{aL}\right)-\bar{U}_{1L}\gamma^\mu d_{1L}-\left(\sum_{\alpha=1}^{3}\bar{N}_{\alpha L}^0\gamma^\mu e_{\alpha L}^-\right)\right)\\
&+X_\mu^+\left(\left(\sum_{a=2}^{3}\bar{u}_{aL}\gamma^\mu D'_{aL}\right)-\bar{U}'_{1L}\gamma^\mu d_{1L}-\left(\sum_{\alpha=1}^{3}\bar{N}'^0_{\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right)\\
&+K_\mu^0\left(\left(\sum_{a=2}^{3}\bar{d}_{aL}\gamma^\mu D_{aL}\right)-\bar{U}_{1L}\gamma^\mu u_{1L}-\left(\sum_{\alpha=1}^{3}\bar{N}_{\alpha L}^0\gamma^\mu \nu_{e\alpha L}\right)\right)\\
&+X_\mu^0\left(\left(\sum_{a=2}^{3}\bar{d}_{aL}\gamma^\mu D'_{aL}\right)-\bar{U}'_{1L}\gamma^\mu u_{1L}-\left(\sum_{\alpha=1}^{3}\bar{N}'^0_{\alpha L}\gamma^\mu \nu_{e\alpha L}\right)\right)\\
&+Y_\mu^0\left(\left(\sum_{a=2}^{3}\bar{D}_{aL}\gamma^\mu D'_{aL}\right)-\bar{U}'_{1L}\gamma^\mu U'_{1L}-\left(\sum_{\alpha=1}^{3}\bar{N}'^0_{\alpha L}\gamma^\mu N_{\alpha L}^0\right)\right)\Bigg]+\text{H.c.}\end{aligned} \quad (4)$$

B. Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$, $J_\mu(Z')$, and $J_\mu(Z'')$ associated with the Hamiltonian

$$H^0=eA^\mu J_\mu(EM)+(g/C_W)Z^\mu J_\mu(Z)+(g'/\sqrt{2})Z'^\mu J_\mu(Z')+(g/2)Z''^\mu J_\mu(Z'')$$

are

$$\begin{aligned}J_\mu(EM)=&\frac{2}{3}\left(\left(\sum_{a=2}^{3}\bar{u}_a\gamma_\mu u_a\right)+\bar{u}_1\gamma_\mu u_1+\bar{U}_1\gamma_\mu U_1+\bar{U}'_1\gamma_\mu U'_1\right)\\
&-\frac{1}{3}\left(\left(\sum_{a=2}^{3}\bar{d}_a\gamma_\mu d_a+\bar{D}_a\gamma_\mu D_a+\bar{D}'_a\gamma_\mu D'_a\right)+\bar{d}_1\gamma_\mu d_1\right)-\sum_{\alpha=1}^{3}\bar{e}_\alpha^-\gamma_\mu e_\alpha^-\\
&=\sum_f q_f\bar{f}\gamma_\mu f,\end{aligned}$$

$$J_\mu(Z)=J_{\mu,L}(Z)-S_W^2 J_\mu(EM),$$

$$J_\mu(Z')=T_W J_\mu(EM)-J_{\mu,L}(Z'),$$

$$J_\mu(Z'')=\sum_{a=2}^{3}(\bar{D}'_{aL}\gamma_\mu D'_{aL}-\bar{D}_{aL}\gamma_\mu D_{aL}) +(-\bar{U}'_{1L}\gamma_\mu U'_{1L}+\bar{U}_{1L}\gamma_\mu U_{1L}) +\sum_{\alpha=1}^{3}(-\bar{N}'^0_{\alpha L}\gamma_\mu N'_{\alpha L}0+\bar{N}^0_{\alpha L}\gamma_\mu N^0_{\alpha L}), \tag{5}$$

where $e=gS_W=g'C_W\sqrt{1-T_W^2/2}>0$ is the electric charge, q_f is the electric charge of the fermion f in units of e and $J_\mu(EM)$ is the electromagnetic current. Notice that the Z''_μ current couples only to exotic fields. The left-handed currents are

$$J_{\mu,L}(Z)=\frac{1}{2}\Bigg[\sum_{a=2}^{3}(\bar{u}_{aL}\gamma_\mu u_{aL}-\bar{d}_{aL}\gamma_\mu d_{aL}) -(\bar{d}_{1L}\gamma_\mu d_{1L}-\bar{u}_{1L}\gamma_\mu u_{1L}) -\sum_{\alpha=1}^{3}(\bar{e}^-_{\alpha L}\gamma_\mu e^-_{\alpha L}-\bar{\nu}_{e\alpha L}\gamma_\mu \nu_{e\alpha L})\Bigg]$$

$$=\sum_f T_{4f}\bar{f}_L\gamma_\mu f_L,$$

$$J_{\mu,L}(Z')=T_W^{-1}\Bigg[\sum_{a=2}^{3}\left(\bar{u}_{aL}\gamma_\mu u_{aL}-\frac{1}{2}\bar{D}_{aL}\gamma_\mu D_{aL}-\frac{1}{2}\overline{D'}_{aL}\gamma_\mu D'_{aL}\right) -\bar{d}_{1L}\gamma_\mu d_{1L}+\frac{1}{2}\bar{U}_{1L}\gamma_\mu U_{1L}+\frac{1}{2}\overline{U'}_{1L}\gamma_\mu U'_{1L} +\sum_{\alpha=1}^{3}\left(-\bar{e}^-_{\alpha L}\gamma_\mu e^-_{\alpha L}+\frac{1}{2}\bar{N}^0_{\alpha L}\gamma_\mu N^0_{\alpha L}+\frac{1}{2}\overline{N'^0}_{\alpha L}\gamma_\mu N'_{\alpha L}0\right)\Bigg]$$

$$=\sum_f T'_{4f}\bar{f}_L\gamma_\mu f_L, \tag{6}$$

where $T_{4f}=Dg(1/2,-1/2,0,0)$ is the third component of the weak isospin and $T'_{4f}=(1/T_W)Dg(1,0,-1/2,-1/2)=(1/T_W)(\lambda_3/2+\lambda_8/\sqrt{3}+\lambda_{15}/\sqrt{6})$ is a convenient 4×4 diagonal matrix, acting both of them on the representation 4 of $SU(4)_L$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with Eqs. (2) and (3).

The couplings of the mass eigenstates Z_1^μ and Z_2^μ are given by

$$H^{NC}=\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\bar{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5)+a_{iR}(f)(1+\gamma_5)]f\}$$
$$=\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\bar{f}\gamma_\mu[g(f)_{iV}-g(f)_{iA}\gamma_5]f\},$$

where

$$a_{1L}(f)=\cos\theta(T_{4f}-q_fS_W^2)+\frac{g'\sin\theta C_W}{g\sqrt{2}}(T'_{4f}-q_fT_W),$$

$$a_{1R}(f)=-q_fS_W\left(\cos\theta S_W+\frac{g'\sin\theta}{g\sqrt{2}}\right),$$

$$a_{2L}(f)=-\sin\theta(T_{4f}-q_fS_W^2)+\frac{g'\cos\theta C_W}{g\sqrt{2}}(T'_{4f}-q_fT_W),$$

$$a_{2R}(f)=q_fS_W\left(\sin\theta S_W-\frac{g'\cos\theta}{g\sqrt{2}}\right), \tag{7}$$

and

$$g(f)_{1V}=\cos\theta\,(T_{4f}-2S_W^2q_f)+\frac{g'\sin\theta}{g\sqrt{2}}(T'_{4f}C_W-2q_fS_W),$$

$$g(f)_{2V}=-\sin\theta(T_{4f}-2S_W^2q_f)+\frac{g'\cos\theta}{g\sqrt{2}}(T'_{4f}C_W-2q_fS_W),$$

$$g(f)_{1A}=\cos\theta T_{4f}+\frac{g'\sin\theta}{g\sqrt{2}}T'_{4f}C_W,$$

$$g(f)_{2A}=-\sin\theta T_{4f}+\frac{g'\cos\theta}{g\sqrt{2}}T'_{4f}C_W. \tag{8}$$

The values of g_{iV}, g_{iA} with $i=1,2$ are listed in Tables I and II.

As we can see, in the limit $\theta=0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

V. FERMION MASSES

The Higgs scalars introduced in Sec. III not only break the symmetry in an appropriate way, but produce the following mass terms for the fermions of the model.

A. Quark masses

For the quark sector we can write the following Yukawa terms:

TABLE I. The $Z_1^\mu \rightarrow \bar{f}f$ couplings.

f	$g(f)_{1V}$	$g(f)_{1A}$
$u_{2,3}$	$\cos\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\sin\theta}{(3C_W^2-1)^{1/2}}\left(1-\frac{7S_W^2}{3}\right)$	$\frac{1}{2}\cos\theta+\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
$d_{2,3}$	$\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right)\cos\theta+\frac{2}{3}\sin\theta S_W^2/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta$
$D_{2,3}$	$\frac{2S_W^2}{3}\cos\theta-\frac{1}{2}\sin\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
$D'_{2,3}$	$\frac{2S_W^2}{3}\cos\theta-\frac{1}{2}\sin\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
d_1	$\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right)\cos\theta-\sin\theta\left(1-\frac{5S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta-\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
u_1	$\cos\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)-\frac{4\sin\theta}{3(3C_W^2-1)^{1/2}}S_W^2$	$\frac{1}{2}\cos\theta$
U_1	$-\frac{4S_W^2\cos\theta}{3}+\sin\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
U'_1	$-\frac{4S_W^2\cos\theta}{3}+\sin\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
$e^-_{1,2,3}$	$\cos\theta(-\frac{1}{2}+2S_W^2)-\frac{\sin\theta}{(3C_W^2-1)^{1/2}}(1-3S_W^2)$	$-\frac{\cos\theta}{2}-\frac{\sin\theta}{(3C_W^2-1)^{1/2}}C_W^2$
$\nu_{1,2,3}$	$\frac{1}{2}\cos\theta$	$\frac{1}{2}\cos\theta$
$N^0_{1,2,3}$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
$N'^0_{1,2,3}$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$

$$
\begin{aligned}
\mathcal{L}_Y^Q = \sum_{\alpha=2}^{3} Q_{\alpha L}^T C \Bigg\{ & \phi_1^* \left(\sum_{\alpha=1}^{3} h_{u\alpha}^a u_{\alpha L}^c + h_U^a U_L^c + h_U^{a'} U_L'^c \right) \\
& + (\phi_2+\phi_3) \left[\sum_{\alpha=1}^{3} h_{\alpha d}^a d_{\alpha L}^c + \sum_{b=2}^{3} (h_{bD}^a D_{bL}^c + h_{bD}^{a'} D_{bL}'^c) \right] \Bigg\} \\
+ Q_{1L}^T C \Bigg\{ & \phi_1 \left[\sum_{\alpha=1}^{3} h_{d\alpha}^1 d_{\alpha L}^c + \sum_{a=2}^{3} (h_{aD}^1 D_{aL}^c + h_{aD}^{1'} D_{aL}'^c) \right] \\
& + (\phi_2^* + \phi_3^*) \left(\sum_{\alpha=1}^{3} h_{u\alpha}^1 u_{\alpha L}^c + h_U^1 U_L^c + h_U^{1'} U_L'^c \right) \Bigg\} + \text{H.c.}
\end{aligned}
$$

where the h's are Yukawa couplings and C is the charge conjugate operator. This Lagrangian produces the following tree level quark masses:

U'_1, D'_2 and D'_3 acquire heavy masses of the order of $V' \gg v$.

U_1, D_2 and D_3 acquire heavy masses of the order of $V \gg v$.

u_3, u_2 and d_1 acquire masses of the order of $v \approx 174$ GeV.

u_1, d_2 and d_3 remain massless at the tree level.

The former mass spectrum is far from being realistic, but it can be improved by implementing the following program:

To introduce a discrete symmetry in order to avoid a tree-level mass for d_1 (and maybe for u_2 too).

To introduce a new Higgs field $\phi_4[1,\bar{4},-1/4]$ which does not acquire VEV but that introduces a quartic coupling $\phi_1^*\phi_2\phi_3\phi_4$ in the Higgs potential in order to generate radiative masses for the ordinary quarks.

To tune the Yukawa couplings in order to obtain the correct mixing between flavors (ordinary and exotic) with the same electric charge.

B. Lepton masses

For the charged leptons we have the following Yukawa terms:

$$
\mathcal{L}_Y^l = \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} h_{\alpha\beta}^e L_{\alpha L}^T C \phi_1 e_{\beta L}^+ + \text{H.c.} \tag{9}
$$

Notice that for $h_{\alpha\beta}^e = h\,\delta_{\alpha\beta}$ we get a mass only for the heavi-

TABLE II. The $Z_2^\mu \to \bar{f} f$ couplings.

f	$g(f)_{2V}$	$g(f)_{2A}$
$u_{2,3}$	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\cos\theta}{(3C_W^2-1)^{1/2}}\left(1-\frac{7S_W^2}{3}\right)$	$-\frac{1}{2}\sin\theta+\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
$d_{2,3}$	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)\sin\theta+\frac{2}{3}\cos\theta S_W^2/(3C_W^2-1)^{1/2}$	$\frac{1}{2}\sin\theta$
$D_{2,3}$	$-\frac{2S_W^2}{3}\sin\theta-\frac{1}{2}\cos\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
$D'_{2,3}$	$-\frac{2S_W^2}{3}\sin\theta-\frac{1}{2}\cos\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
d_1	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)\sin\theta-\cos\theta\left(1-\frac{5S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$\frac{1}{2}\sin\theta-\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
u_1	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)-\frac{4\cos\theta}{3(3C_W^2-1)^{1/2}}S_W^2$	$-\frac{1}{2}\sin\theta$
U_1	$\frac{4S_W^2\sin\theta}{3}+\cos\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
U'_1	$\frac{4S_W^2\sin\theta}{3}+\cos\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
$e^-_{1,2,3}$	$\sin\theta(\frac{1}{2}-2S_W^2)-\frac{\cos\theta}{(3C_W^2-1)^{1/2}}(1-3S_W^2)$	$\frac{\sin\theta}{2}-\frac{\cos\theta}{(3C_W^2-1)^{1/2}}C_W^2$
$\nu_{1,2,3}$	$-\frac{1}{2}\sin\theta$	$-\frac{1}{2}\sin\theta$
$N^0_{1,2,3}$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
$N'^0_{1,2,3}$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$

est lepton (the τ). So, in the context of this model the masses for the charged leptons can be generated in a consistent way, with the masses for e^- and μ^- suppressed by differences of Yukawa couplings.

The neutral leptons remain massless as far as we use only the original fields introduced in Sec. II. But as mentioned earlier, we may introduce one or more Weyl singlet states $N^0_{L,b}$, $b=1,2,\ldots$ which may implement the appropriate neutrino oscillations [3].

VI. CONSTRAINS ON THE $(Z^\mu - Z'^\mu)$ MIXING ANGLE AND THE Z_2^μ MASS

To bound $\sin\theta$ and M_{Z_2} we use parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table III.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$ is

$$\Gamma(Z_1^\mu \to f\bar{f}) = \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\left\{\frac{3\beta-\beta^3}{2}[g(f)_{1V}]^2 + \beta^3[g(f)_{1A}]^2\right\}(1+\delta_f)R_{EW}R_{QCD}, \quad (10)$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C=1$ for leptons while for quarks $N_C=3(1+\alpha_s/\pi+1.405\alpha_s^2/\pi^2-12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths see Ref. [7]); R_{EW} are the electroweak corrections which include the leading order QED corrections given by $R_{QED}=1+3\alpha/(4\pi)$. R_{QCD} are further QCD corrections (for a comprehensive review see Ref. [8] and references therein), and $\beta=\sqrt{1-4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}[-m_t^2/(2M_{Z_1}^2)+1/5]$ [9]. The ρ parameter can be expanded as $\rho=1+\delta\rho_0+\delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0 \approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree level contribution due to the $(Z_\mu - Z'_\mu)$ mixing which can be parametrized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2-1)\sin^2\theta$. Fi-

TABLE III. Experimental data and SM values for the parameters.

	Experimental results	SM
Γ_Z(GeV)	2.4952 ± 0.0023	2.4966 ± 0.0016
Γ(had) (GeV)	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.019 ± 0.027
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21664 ± 0.00068	0.21569 ± 0.00016
R_c	0.1729 ± 0.0032	0.17230 ± 0.00007
Q_W^{Cs}	$-72.65\pm0.28\pm0.34$	-73.10 ± 0.03
M_{Z_1} (GeV)	91.1872 ± 0.0021	91.1870 ± 0.0021

nally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which are listed in Table I.

In what follows we are going to use the experimental values [10]: $M_{Z_1}=91.188$ GeV, $m_t=174.3$ GeV, $\alpha_s(m_Z)=0.1192$, $\alpha(m_Z)^{-1}=127.938$, and $\sin^2\theta_W=0.2333$. The experimental values are introduced using the definitions $R_\eta\equiv\Gamma(\eta\eta)/\Gamma(\text{hadrons})$ for $\eta=e,\mu,\tau,b,c$.

As a first result notice from Table I that our model predicts $R_e=R_\mu=R_\tau$, in agreement with the experimental results in Table III.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W=-2[(2Z+N)c_{1u}+(Z+2N)c_{1d}], \qquad (11)$$

where $c_{1q}=2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the cesium atom is given by [11] $Q_W(^{133}_{55}\text{Cs})=-73.09\pm0.04+\Delta Q_W$, where the contribution of new physics is included in ΔQ_W which can be written as [12]

$$\Delta Q_W=\left[\left(1+4\frac{S_W^4}{1-2S_W^2}\right)Z-N\right]\delta\rho_V+\Delta Q_W'. \qquad (12)$$

The term $\Delta Q_W'$ is model dependent and it can be obtained for our model by using $g(e)_{1A}$ and $g(q)_{1V}$ from Table I. The value we obtain is

$$\Delta Q_W'=(8.29Z+16.14N)\sin\theta+(11.64Z+14.47N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \qquad (13)$$

The discrepancy between the SM and the experimental data for ΔQ_W is given by [13]

$$\Delta Q_W=Q_W^{exp}-Q_W^{SM}=1.03\pm0.44, \qquad (14)$$

which is 2.3 σ away from the SM predictions.

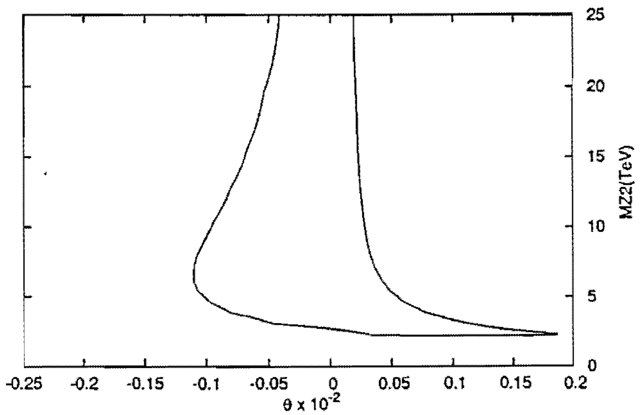

FIG. 1. Contour plot displaying the allowed region for θ vs M_{Z_2} at 95% C.L.

Introducing the expressions for Z pole observables in Eq. (10), with ΔQ_W in terms of new physics in Eq. (12) and using experimental data from LEP, SLC and atomic parity violation (see Table III), we do a χ^2 fit and we find the best allowed region in the $(\theta-M_{Z_2})$ plane at 95% confidence level (C.L.). In Fig. 1 we display this region which gives us the constraints

$$-0.0011\leq\theta\leq0.0019, \quad 2\ \text{TeV}\leq M_{Z_2}. \qquad (15)$$

As we can see the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [14]. From our analysis we can also see that for $|\theta|\to0$, M_{Z_2} peaks at a finite value larger than 100 TeV which still copes with the experimental constraints on the ρ parameter.

VII. CONCLUSIONS

We have presented an anomaly-free model based on the local gauge group $SU(3)_c\otimes SU(4)_L\otimes U(1)_X$. We break the gauge symmetry down to $SU(3)_c\otimes U(1)_Q$ and at the same time give masses to the fermion fields in the model in a consistent way by using three different Higgs scalars ϕ_i, $i=1,2,3$ which set two different mass scales: $V\sim V'\gg v=174$ GeV. By using experimental results we bound the mixing angle θ between the SM neutral current and a new one to be $-0.0011<\theta<0.0019$ and the lowest bound for M_{Z_2} is 2 TeV$\leq M_{Z_2}$.

Our model includes four exotic down type quarks D_a, D_a', $a=2,3$ of electric charge $-1/3$ and two exotic up quarks U_1, U_1' of electric charge 2/3. The six exotic quarks acquire large masses of the order of $V\simeq V'\gg v=174$ GeV and are useful in two ways: first they mix with the ordinary up and down quarks in the three families with a mixing that can be used in order to produce a consistent mass spectrum (masses and mixings) for ordinary quarks; second, they can be used in order to implement the so-called little Higgs mechanism [5].

Notice also the consistence of our model in the charged lepton sector. Not only it predicts the correct ratios R_η, η

$=e,\mu,\tau$ in the Z decays, but the model also allows for a consistent mass pattern of the particles, which do not include leptons with exotic electric charges.

In the main body of this paper we have analyzed an specific model based on the 3-4-1 gauge structure. This model is just one of a large variety of models based on the same gauge structure. A systematic analysis of models without exotic electric charges with the same gauge structure is presented in the Appendix at the end of the paper. A phenomenological analysis for all those model can be done, but we think it is not profitable since all of them must produce similar results at low energies.

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APPENDIX

In what follows we present a systematic analysis of models without exotic electric charges, based on the local gauge structure $SU(3)_c\otimes SU(4)_L\otimes U(1)_X$.

We assume that the electroweak group is $SU(4)_L \otimes U(1)_X\supset SU(3)_L\otimes U(1)_Z\supset SU(2)_L\otimes U(1)_Y$, where the gauge structure $SU(3)_L\otimes U(1)_Z$ refers to the one presented in Ref. [6]. We also assume that the left-handed quarks (color triplets), left-handed leptons (color singlets) and scalars, transform either under the 4 or the $\bar{4}$ fundamental representations of $SU(4)_L$. Two classes of models will be discussed: one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the several families. As in the SM, $SU(3)_c$ is vectorlike.

The most general expression for the electric charge generator in $SU(4)_L\otimes U(1)_X$ is a linear combination of the four diagonal generators of the gauge group

$$Q=aT_{3L}+\frac{1}{\sqrt{3}}bT_{8L}+\frac{1}{\sqrt{6}}cT_{15L}+XI_4, \tag{A1}$$

where $T_{iL}=\lambda_{iL}/2$, being λ_{iL} the Gell-Mann matrices for $SU(4)_L$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}$, $I_4=Dg(1,1,1,1)$ is the diagonal 4×4 unit matrix, and a, b and c are free parameters to be fixed next. Notice that we can absorb an eventual coefficient for X in its definition.

If we assume that the usual isospin $SU(2)_L$ of the SM is such that $SU(2)_L\subset SU(4)_L$, then $a=1$ and we have just a two-parameter set of models, all of them characterized by the values of b and c. So, Eq. (A1) allows for an infinite number of models in the context of the 3-4-1 theory, each one associated to particular values of the parameters b and c, with characteristic signatures that make them different from each other.

There are a total of 24 gauge bosons in the gauge group under consideration, 15 of them associated with $SU(4)_L$ which can be written as

$$\frac{1}{2}\lambda_\alpha A^\alpha_\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} D^0_{1\mu} & W^+_\mu & K^{(b+1)/2}_\mu & X^{(3+b+2c)/6}_\mu \\ W^-_\mu & D^0_{2\mu} & K^{(b-1)/2}_\mu & X^{(-3+b+2c)/6}_\mu \\ K^{-(b+1)/2}_\mu & \bar{K}^{-(b-1)/2}_\mu & D^0_{3\mu} & Y^{-(b-c)/3}_\mu \\ X^{-(3+b+2c)/6}_\mu & \bar{X}^{(3-b-2c)/6}_\mu & \bar{Y}^{(b-c)/3}_\mu & D^0_{4\mu} \end{pmatrix}, \tag{A2}$$

where $D^\mu_1=A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, $D^\mu_2=-A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$; $D^\mu_3=-2A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, and $D^\mu_4=-3A^\mu_{15}/\sqrt{12}$. The upper indices in the gauge bosons in the former expression stand for the electric charge of the corresponding particle, some of them functions of the b and c parameters as they should be. Notice that if we demand for gauge bosons with electric charges $0,\pm1$ only, there are not more than four different possibilities for the simultaneous values of b and c; they are: $b=c=1$; $b=c=-1$; $b=1,c=-2$, and $b=-1,c=2$.

Now, contrary to the SM where only the Abelian $U(1)_Y$ factor is anomalous, in the 3-4-1 theory both, $SU(4)_L$ and $U(1)_X$ are anomalous [$SU(3)_c$ is vectorlike]. So, special combinations of multiplets must be used in each particular model in order to cancel the possible anomalies, and obtain renormalizable models. The triangle anomalies we must take care of are: $[SU(4)_L]^3$, $[SU(3)_c]^2U(1)_X$, $[SU(4)_L]^2U(1)_X$, $[grav]^2U(1)_X$ and $[U(1)_X]^3$.

Now let us see how the charge operator in Eq. (A1) acts on the representations 4 and $\bar{4}$ of $SU(4)_L$:

$$Q[4]=Dg\left(\frac{1}{2}+\frac{b}{6}+\frac{c}{12}+X,-\frac{1}{2}+\frac{b}{6}+\frac{c}{12}+X,\right.$$
$$\left.-\frac{2b}{6}+\frac{c}{12}+X,-\frac{3c}{12}+X\right),$$

$$Q[\bar{4}]=Dg\left(-\frac{1}{2}-\frac{b}{6}-\frac{c}{12}+X,\frac{1}{2}-\frac{b}{6}-\frac{c}{12}+X,\right.$$
$$\left.\frac{2b}{6}-\frac{c}{12}+X,\frac{3c}{12}+X\right).$$

TABLE IV. Anomalies for sets with values $b=c=1$.

Anomaly	S_1^q	S_2^q	S_3^l	S_4^l	S_5^l	S_6^l
$[U(1)_X]^3$	$-9/16$	$-27/16$	$21/16$	$-15/16$	$15/16$	$-21/16$
$[SU(4)_L]^2U(1)_X$	$-1/4$	$5/4$	$-3/4$	$1/4$	$-1/4$	$3/4$
$[SU(4)_L]^3$	3	-3	1	1	-1	-1

Notice that, if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of 4 and $\bar{4}$ (or $\bar{4}$ and 4), do not allow for electrically charged antiparticles in the two lower components of the multiplets [antiquarks violate $SU(3)_c$ and e^+,μ^+ and τ^+ violate lepton number at the tree level] and forbid the presence of exotic electric charges in the possible models, then the electric charge of the third and fourth components in 4 and $\bar{4}$ must be equal either to the charge of the first and/or second component, which in turn implies that b and c can take only the four sets of values stated above. So, these four sets of values for b and c are necessary and sufficient conditions in order to exclude exotic electric charges in the fermion sector too.

A further analysis also shows that models with $b=c=-1$ are equivalent, via charge conjugation, to models with $b=c=1$. Similarly, models with $b=-1$, $c=2$ are equivalent to models with $b=1$, $c=-2$. So, with the constraints imposed, we have only two different sets of models; those for $b=c=1$ and those for $b=1$, $c=-2$.

1. Models for $b=c=1$

First let us define the following complete sets of spin 1/2 Weyl spinors (complete in the sense that each set contains its own charged antiparticles):

$$S_1^q=\{(u,d,D,D')_L\sim[3,4,-\tfrac{1}{12}],\ u_L^c\sim[\bar{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ D_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ D_L'^c\sim[\bar{3},1,\tfrac{1}{3}]\}.$$

$$S_2^q=\{(d,u,U,U')_L\sim[3,\bar{4},\tfrac{5}{12}],\ u_L^c\sim[\bar{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ U_L^c\sim[\bar{3},1,-\tfrac{2}{3}],\ U_L'^c\sim[\bar{3},1,-\tfrac{2}{3}]\}.$$

$$S_3^l=\{(\nu_e^0,e^-,E^-,E'^-)_L\sim[1,4,-\tfrac{3}{4}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1],\ E_L'^+\sim[1,1,1]\}.$$

$$S_4^l=\{(E^+,N_1^0,N_2^0,N_3^0)_L\sim[1,4,\tfrac{1}{4}],\ E_L^-\sim[1,1,-1]\}.$$

$$S_5^l=\{(e^-,\nu_e^0,N^0,N'^0)_L\sim[1,\bar{4},-\tfrac{1}{4}],\ e_L^+\sim[1,1,1]\}.$$

$$S_6^l=\{(N,E_1^+,E_2^+,E_3^+)_L\sim[1,\bar{4},\tfrac{3}{4}],\ E_{1L}^-\sim[1,1,-1],$$
$$E_{2L}^-\sim[1,1,-1],\ E_{3L}^-\sim[1,1,-1]\}.$$

Due to the fact that each set includes charged particles together with their corresponding antiparticles, and since $SU(3)_c$ is vectorlike, the anomalies $[\mathrm{grav}]^2U(1)_X$, $[SU(3)_c]^3$ and $[SU(3)_c]^2U(1)_X$ automatically vanish. So, we only have to take care of the remaining three anomalies whose values are shown in Table IV.

Several anomaly free models can be constructed from this table. Let us see.

a. Three family models

We found two three family structures which are:

Model $A=2S_1^q\oplus S_2^q\oplus 3S_5^l$. (The model analyzed in the main text.)

Model $B=S_1^q\oplus 2S_2^q\oplus 3S_3^l$.

b. Two family models

We find only one two family structure given by: Model $C=S_1^q\oplus S_2^q\oplus S_3^l\oplus S_5^l$.

c. One family models

A one family model can not be directly extracted from S_i, $i=1,2,\ldots,6$, but we can check that the following particular arrangement is an anomaly free one family structure: Model $D=S_1^q\oplus(e^-,\nu_e^0,N^0,N'^0)_L\oplus(E_1^-,N_1^0,N_2^0,N_3^0)_L\oplus(N_4^0,E_1^+,e^+,E_2^+)_L\oplus E_2^-$. As can be checked, this model reduces to the model in Ref. [15] for the breaking chain $SU(4)_L\otimes U(1)_X\rightarrow SU(3)_L\otimes U(1)_\alpha\otimes U(1)_X\rightarrow SU(3)_L\otimes U(1)_Z$, for the value $\alpha=1/12$. In an analogous way, other one family models with more exotic charged leptons can also be constructed.

2. Models for $b=1,c=-2$

As in the previous case, let us define the following complete sets of spin 1/2 Weyl spinors:

$$S_1'^q=\{(u,d,D,U)_L\sim[3,4,\tfrac{1}{6}],\ u_L^c\sim[\bar{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ D_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ U_L^c\sim[\bar{3},1,-\tfrac{2}{3}]\}.$$

$$S_2'^q=\{(d,u,U,D)_L\sim[3,\bar{4},\tfrac{1}{6}],\ u_L^c\sim[\bar{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\bar{3},1,\tfrac{1}{3}],\ U_L^c\sim[\bar{3},1,-\tfrac{2}{3}],\ D_L^c\sim[\bar{3},1,\tfrac{1}{3}]\}.$$

$$S_3'^l=\{(\nu_e^0,e^-,E^-,N^0)_L\sim[1,4,-\tfrac{1}{2}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1]\}.$$

$$S_4'^l=\{(e^-,\nu_e^0,N^0,E^-)_L\sim[1,\bar{4},-\tfrac{1}{2}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1]\}.$$

TABLE V. Anomalies for sets with values $b=1$, $c=-2$.

Anomaly	S_1^q	S_2^q	S_3^l	S_4^l	S_5^l	S_6^l
$[U(1)_X]^3$	$-3/2$	$-3/2$	$3/2$	$3/2$	$-3/2$	$-3/2$
$[SU(4)_L]^2U(1)_X$	$1/2$	$1/2$	$-1/2$	$-1/2$	$1/2$	$1/2$
$[SU(4)_L]^3$	3	-3	1	-1	1	-1

$$S_5'^l=\{(E^+,N_1^0,N_2^0,e^+)_L\sim[1,4,\tfrac{1}{2}],\ E_L^-\sim[1,1,-1],\ e_L^-\sim[1,1,-1]\}.$$

$$S_6'^l=\{(N_3^0,E^+,e^+,N_4^0)_L\sim[1,\bar{4},\tfrac{1}{2}],\ E_L^-\sim[1,1,-1],\ e_L^-\sim[1,1,-1]\}.$$

For the former sets the anomalies $[\text{grav}]^2U(1)_X$, $[SU(3)_c]^3$ and $[SU(3)_c]^2U(1)_X$ vanish. The other anomalies are shown in Table V. Again, several anomaly free models can be constructed from this table. Let us see.

a. Three family models

We found two three family structures which are:
Model E$=2S_1'^q\oplus S_2'^q\oplus 3S_4'^l$.
Model F$=S_1'^q\oplus 2S_2'^q\oplus 3S_3'^l$.

b. Two family models

We find again only one two family structure given by: Model G$=S_1'^q\oplus S_2'^q\oplus S_3'^l\oplus S_4'^l$.

c. One family models

Two one family models can be constructed using S_i', $i=1,\ldots,6$. They are:

Model H$=S_2'^q\oplus 2S_3'^l\oplus S_5'^l$.

Model I$=S_1'^q\oplus 2S_4'^l\oplus S_6'^l$.

To conclude this appendix let us mention that for the values of the parameters b and c used in our analysis, many more anomaly-free models can be constructed, all of them featuring the SM phenomenology at energies below 100 GeV. Model A, discussed in the main text, is just one example.

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THE EUROPEAN PHYSICAL JOURNAL C

$SU(3)_c \otimes SU(4)_{\mathrm{L}} \otimes U(1)_X$ model for three families

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Abstract. An extension of the standard model to the local gauge group $SU(3)_c \otimes SU(4)_{\mathrm{L}} \otimes U(1)_X$ as a three-family model is presented. The model does not contain exotic electric charges and we obtain a consistent mass spectrum by introducing an anomaly-free discrete Z_2 symmetry. The neutral currents coupled to all neutral vector bosons in the model are studied. By using experimental results from the CERN LEP, SLAC Linear Collider and atomic parity violation data we constrain the mixing angle between two of the neutral currents in the model and the mass of the additional neutral gauge bosons to be $-0.0032 \leq \sin\theta \leq 0.0031$ and $0.67\,\mathrm{TeV} \leq M_{Z_2} \leq 6.1\,\mathrm{TeV}$ at 95% C.L., respectively.

1 Introduction

The standard model (SM), based on the local gauge group $SU(3)_c \otimes SU(2)_{\mathrm{L}} \otimes U(1)_Y$ [1], can be extended in several different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitutes its simplest extension and has profound consequences, as the implementation of the see-saw mechanism, and the enlarging of the possible number of local abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one Higgs representation, and, third, by enlarging the local gauge group. In this last direction $SU(4)_{\mathrm{L}} \otimes U(1)_X$ as a flavor group has been considered before in the literature [2–4] which, among its best features, provides us with an alternative to the problem of the number N_f of families, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, N_c being the number of colors of $SU(3)_c$ (also known as QCD). Moreover, this gauge structure has been used recently in order to implement the so-called little Higgs mechanism [4].

The analysis of the local gauge structure $SU(3)_c \otimes SU(4)_{\mathrm{L}} \otimes U(1)_X$ (hereafter the 3-4-1 group) presented in the appendix of [3] shows that we may write the most general electric charge operator for this group as

$$Q = aT_{3\mathrm{L}} + \frac{b}{\sqrt{3}}T_{8\mathrm{L}} + \frac{c}{\sqrt{6}}T_{15\mathrm{L}} + XI_4, \qquad (1)$$

where a, b and c are free parameters, $T_{i\mathrm{L}} = \lambda_{i\mathrm{L}}/2$, with $\lambda_{i\mathrm{L}}$ the Gell-Mann matrices for $SU(4)_{\mathrm{L}}$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$, and $I_4 = \mathrm{Dg}(1,1,1,1)$ is the diagonal 4×4 unit matrix. The X values are fixed by anomaly cancellation of the fermion content in the possible models and an eventual coefficient for XI_4 can be absorbed in the X hypercharge definition. The free parameters a, b and c fix the gauge boson structure of the electroweak sector $[SU(4)_{\mathrm{L}} \otimes U(1)_X]$, and also the electroweak charges of the scalar representations which are fully determined by the symmetry breaking pattern implemented. In particular $a = 1$ gives the usual isospin of the known electroweak interactions, with b and c remaining as free parameters, producing an infinite plethora of possible models.

Restricting the particle content of the model to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are generated [3]. In particular, the restriction to ordinary electric charges, in the fermion, scalar and gauge boson sectors, allows only for two different cases for the simultaneous values of the parameters b and c, namely: $b = c = 1$ and $b = 1, c = -2$, which become a convenient classification scheme for these types of models. Models in the first class differ from those in the second one not only in their fermion content but also in their gauge and scalar boson sectors. Four of the identified models without exotic electric charges are three-family models in the sense that anomalies cancel among the three families of quarks and leptons in a non-trivial fashion. Two of them are models for which $b = c = 1$, and one of them has been analyzed in [3]. The other two models belong to the class for which $b = 1, c = -2$ and one of them, the so-called "Model **E**" in the appendix of [3], will be studied in this paper. It is worth noticing that in the four different models at least one of the three families is treated differently.

This paper is organized as follows. In the next section we describe the fermion content of the particular model we are going to study. In Sect. 3 we introduce the scalar sector. In Sect. 4 we study the gauge boson sector, paying special attention to the neutral currents present in the model and their mixing. In Sect. 5 we analyze the fermion mass spectrum. In Sect. 6 we use experimental results in

Table 1. Anomaly-free fermion structure of Model **E** from [3]

$Q_{1L} = \begin{pmatrix} d_1 \\ u_1 \\ U_1 \\ D_1 \end{pmatrix}_L$	d^c_{1L}	u^c_{1L}	U^c_{1L}	D^c_{1L}
$[3, 4^*, \frac{1}{6}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$
$Q_{jL} = \begin{pmatrix} u_j \\ d_j \\ D_j \\ U_j \end{pmatrix}_L$	u^c_{jL}	d^c_{jL}	D^c_{jL}	U^c_{jL}
$[3, 4, \frac{1}{6}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$
$L_{\alpha L} = \begin{pmatrix} e^-_\alpha \\ \nu_{e\alpha} \\ N^0_\alpha \\ E^-_\alpha \end{pmatrix}_L$	$e^+_{\alpha L}$	$E^+_{\alpha L}$.		
$[1, 4^*, -\frac{1}{2}]$	$[1, 1, 1]$	$[1, 1, 1]$		

order to constrain the mixing angle between two of the neutral currents in the model and the mass scale of the new neutral gauge bosons. In the last section we summarize the model and state our conclusions.

2 The fermion content of the model

In what follows we assume that the electroweak gauge group is $SU(4)_L \otimes U(1)_X$ which contains $SU(2)_L \otimes U(1)_Y$ as a subgroup. We will consider the case of a non-universal hypercharge X in the quark sector, which implies anomaly cancellation among the three families in a non-trivial fashion.

Here we are interested in studying the phenomenology of three-family models without exotic electric charges and with values $b = 1, c = -2$ for the parameters in the electric charge generator in (1). As an example we take Model **E** of [3] for which the electric charge operator is given by $Q = T_{3L} + T_{8L}/\sqrt{3} - 2T_{15L}/\sqrt{6} + XI_4$. This model has the anomaly-free fermion structure as given in Table 1. where $j = 2, 3$ and $\alpha = 1, 2, 3$ are two- and three-family indexes, respectively. The numbers in parentheses refer to the $[SU(3)_C, SU(4)_L, U(1)_X]$ quantum numbers, respectively. Notice that, if needed, the lepton structure of the model can be augmented with an undetermined number of neutral Weyl singlet states $N^0_{L,n} \sim [1, 1, 0]$, $n = 1, 2, \ldots,$ without violating our assumptions, neither the anomaly constraint relations, because singlets with no X charges are as good as not being present as far as anomaly cancellation is concerned.

3 The scalar sector

Our aim is to break the symmetry, following the pattern

$$\begin{aligned} &SU(3)_c \otimes SU(4)_L \otimes U(1)_X \\ &\to SU(3)_c \otimes SU(3)_L \otimes U(1)_X \\ &\to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ &\to SU(3)_c \otimes U(1)_Q, \end{aligned}$$

where $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ refers to the so-called 3-3-1 structure introduced in [5]. At the same time we want to give masses to the fermion fields in the model. With this in mind we introduce the following four Higgs scalars: $\phi_1[1, 4^*, -1/2]$ with a vacuum expectation value (VEV) aligned in the direction $\langle\phi_1\rangle = (0, v, 0, 0)^T$; $\phi_2[1, 4^*, -1/2]$ with a VEV aligned as $\langle\phi_2\rangle = (0, 0, V, 0)^T$; $\phi_3[1, 4, -1/2]$ with a VEV aligned in the direction $\langle\phi_3\rangle = (v', 0, 0, 0)^T$, and $\phi_4[1, 4, -1/2]$ with a VEV aligned as $\langle\phi_4\rangle = (0, 0, 0, V')^T$, with the hierarchy $V \sim V' >> \sqrt{v^2 + v'^2} \simeq 174$ GeV (the electroweak breaking scale).

4 The gauge boson sector

In the model there are a total of 24 gauge bosons: One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 15 gauge fields associated with $SU(4)_L$ which, for $b = 1$ and $c = -2$, can be written as

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} & X^{0\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} & X^{-\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 & Y^{-\mu} \\ \bar{X}^{0\mu} & X^{+\mu} & Y^{+\mu} & D^\mu_4 \end{pmatrix},$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, $D^\mu_3 = -2A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, and $D^\mu_4 = -3A^\mu_{15}/\sqrt{12}$.

After breaking the symmetry with $\langle\phi_1\rangle + \langle\phi_2\rangle + \langle\phi_3\rangle + \langle\phi_4\rangle$ and using for the covariant derivative for 4-plets $\mathrm{i}D^\mu = \mathrm{i}\partial^\mu - g\lambda_\alpha A^\mu_\alpha/2 - g'XB^\mu$, where g and g' are the $SU(4)_L$ and $U(1)_X$ gauge coupling constants respectively, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm} = g^2(v^2 + v'^2)/2$, $M^2_{K^\pm} = g^2(v'^2 + V^2)/2$, $M^2_{X^\pm} = g^2(v^2 + V'^2)/2$, $M^2_{Y^\pm} = g^2(V^2 + V'^2)/2$, $M^2_{K^0(\bar{K}^0)} = g^2(v^2 + V^2)/2$, and $M^2_{X^0(\bar{X}^0)} = g^2(v'^2 + V'^2)/2$. Since $W^\pm$ does not mix with the other charged bosons we have that $\sqrt{v^2 + v'^2} \approx 174$ GeV as mentioned in the previous section.

For the four neutral gauge bosons we get mass terms of the form

$$\begin{aligned} M = \frac{g^2}{2} \Bigg\{ & V^2 \left(\frac{g'B^\mu}{g} - \frac{2A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}} \right)^2 \\ & + V'^2 \left(\frac{g'B^\mu}{g} + \frac{3A^\mu_{15}}{\sqrt{6}} \right)^2 \\ & + v'^2 \left(A^\mu_3 + \frac{A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}} - \frac{g'B^\mu}{g} \right)^2 \\ & + v^2 \left(\frac{g'B^\mu}{g} - A^\mu_3 + \frac{A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}} \right)^2 \Bigg\}. \end{aligned}$$

M is a 4×4 matrix with a zero eigenvalue corresponding to the photon. Once the photon field has been identified, there remains a 3×3 mass matrix for three neutral gauge bosons, Z^μ, Z'^μ and Z''^μ. Since we are interested now in the low energy phenomenology of our model, we can choose $V = V'$ in order to simplify matters. Also, the mixing between the three neutral gauge bosons can be further simplified by choosing $v' = v$. For this particular case the field $Z''^\mu = 2A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{3}$ decouples from the other two and acquires a squared mass $(g^2/2)(V^2 + v^2)$. By diagonalizing the remaining 2×2 mass matrix we get two other physical neutral gauge bosons, which are defined through the mixing angle θ between Z_μ, Z'_μ:

$$\begin{aligned} Z_1^\mu &= Z_\mu \cos\theta + Z'_\mu \sin\theta\ , \\ Z_2^\mu &= -Z_\mu \sin\theta + Z'_\mu \cos\theta, \end{aligned}$$

where

$$\tan(2\theta) = \frac{S_W^2 \sqrt{C_{2W}}}{(1+S_W^2)^2 + \frac{V^2}{v^2} C_W^4 - 2}. \qquad (2)$$

$S_W = g'/\sqrt{2g'^2 + g^2}$ and C_W are the sine and cosine of the electroweak mixing angle, respectively, and $C_{2W} = C_W^2 - S_W^2$.

The photon field A^μ and the fields Z_μ and Z'_μ are given by

$$\begin{aligned} A^\mu &= S_W A_3^\mu \\ &\quad + C_W \left[\frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right], \\ Z^\mu &= C_W A_3^\mu \\ &\quad - S_W \left[\frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right], \\ Z'^\mu &= \frac{1}{\sqrt{3}} (1 - T_W^2)^{1/2} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) - T_W B^\mu. \end{aligned} \qquad (3)$$

We can also identify the Y hypercharge associated with the SM abelian gauge boson as

$$Y^\mu = \frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu. \qquad (4)$$

4.1 Charged currents

The Hamiltonian for the charged currents in the model is given by

$$\begin{aligned} H^{CC} &= \frac{g}{\sqrt{2}} \\ &\times \Bigg\{ W_\mu^+ \left[\left(\sum_{j=2}^{3} \bar{u}_{aL} \gamma^\mu d_{aL} \right) - \bar{u}_{1L} \gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3} \bar{\nu}_{e\alpha L} \gamma^\mu e_{\alpha L}^- \right) \right] \\ &+ K_\mu^+ \left[\left(\sum_{j=2}^{3} \bar{u}_{aL} \gamma^\mu D_{aL} \right) - \bar{U}_{1L} \gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3} \bar{N}_{\alpha L}^0 \gamma^\mu e_{\alpha L}^- \right) \right] \\ &+ X_\mu^+ \left[\left(\sum_{j=2}^{3} \bar{U}_{aL} \gamma^\mu d_{aL} \right) - \bar{u}_{1L} \gamma^\mu D_{1L} - \left(\sum_{\alpha=1}^{3} \bar{\nu}_{e\alpha L} \gamma^\mu E_{\alpha L}^- \right) \right] \\ &+ Y_\mu^+ \left[\left(\sum_{j=2}^{3} \bar{U}_{aL} \gamma^\mu D_{aL} \right) - \bar{U}_{1L} \gamma^\mu D_{1L} - \left(\sum_{\alpha=1}^{3} \bar{N}_{\alpha L}^0 \gamma^\mu E_{\alpha L}^- \right) \right] \\ &+ K_\mu^0 \left[\left(\sum_{j=2}^{3} \bar{d}_{aL} \gamma^\mu D_{aL} \right) - \bar{U}_{1L} \gamma^\mu u_{1L} - \left(\sum_{\alpha=1}^{3} \bar{N}_{\alpha L}^0 \gamma^\mu \nu_{e\alpha L} \right) \right] \\ &+ X_\mu^0 \left[\left(\sum_{j=2}^{3} \bar{u}_{aL} \gamma^\mu U_{aL} \right) - \bar{D}_{1L} \gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3} \bar{E}_{\alpha L}^- \gamma^\mu e_{\alpha L}^- \right) \right] \Bigg\} \\ &+ \text{h.c.} \end{aligned}$$

4.2 Neutral currents

The neutral currents $J_\mu(\text{EM})$, $J_\mu(Z)$, $J_\mu(Z')$, and $J_\mu(Z'')$ associated with the Hamiltonian

$$\begin{aligned} H^0 &= eA^\mu J_\mu(\text{EM}) + (g/C_W) Z^\mu J_\mu(Z) \\ &\quad + (g') Z'^\mu J_\mu(Z') + (g/(2\sqrt{2})) Z''^\mu J_\mu(Z''), \end{aligned}$$

are

$$\begin{aligned} &J_\mu(\text{EM}) \\ &= \frac{2}{3} \left[\sum_{j=2}^{3} (\bar{u}_a \gamma_\mu u_a + \bar{U}_a \gamma_\mu U_a) + \bar{u}_1 \gamma_\mu u_1 + \bar{U}_1 \gamma_\mu U_1 \right] \\ &\quad - \frac{1}{3} \left[\sum_{j=2}^{3} (\bar{d}_a \gamma_\mu d_a + \bar{D}_a \gamma_\mu D_a) + \bar{d}_1 \gamma_\mu d_1 + \bar{D}_1 \gamma_\mu D_1 \right] \\ &\quad - \sum_{\alpha=1}^{3} \bar{e}_\alpha^- \gamma_\mu e_\alpha^- - \sum_{\alpha=1}^{3} \bar{E}_\alpha^- \gamma_\mu E_\alpha^- \\ &= \sum_f q_f \bar{f} \gamma_\mu f, \\ &J_\mu(Z) = J_{\mu,L}(Z) - S_W^2 J_\mu(\text{EM}), \\ &J_\mu(Z') = J_{\mu,L}(Z') - T_W J_\mu(\text{EM}), \\ &J_\mu(Z'') \\ &= \sum_{a=2}^{3} (\bar{u}_{aL} \gamma_\mu u_{aL} + \bar{d}_{aL} \gamma_\mu d_{aL} - \bar{D}_{aL} \gamma_\mu D_{aL} - \bar{U}_{aL} \gamma_\mu U_{aL}) \\ &\quad - \bar{d}_{1L} \gamma_\mu d_{1L} - \bar{u}_{1L} \gamma_\mu u_{1L} + \bar{U}_{1L} \gamma_\mu U_{1L} + \bar{D}_{1L} \gamma_\mu D_{1L} \\ &\quad + \sum_{\alpha=1}^{3} (-\bar{e}_{\alpha L}^- \gamma_\mu e_{\alpha L}^- - \bar{\nu}_{e\alpha L} \gamma_\mu \nu_{e\alpha L} \\ &\quad + \bar{N}_{\alpha L}^0 \gamma_\mu N_{\alpha L}^0 + \bar{E}_{\alpha L}^- \gamma_\mu E_{\alpha L}^-), \end{aligned} \qquad (5)$$

where $e = gS_W = g'C_W\sqrt{1 - T_W^2} > 0$ is the electric charge, q_f is the electric charge of the fermion f in units of e, and $J_\mu(\text{EM})$ is the electromagnetic current. Note from

$J_\mu(Z'')$ that, notwithstanding the extra neutral gauge boson, Z''_μ does not mix with Z_μ or Z'_μ (for the particular case $V = V'$ and $v = v'$); it still couples to ordinary fermions.

The left-handed currents are

$$\begin{aligned}J_{\mu,\mathrm{L}}(Z) &= \frac{1}{2}\left[\sum_{j=2}^{3}(\bar{u}_{a\mathrm{L}}\gamma_\mu u_{a\mathrm{L}} - \bar{d}_{a\mathrm{L}}\gamma_\mu d_{a\mathrm{L}}) \right.\\ &\quad -(\bar{d}_{1\mathrm{L}}\gamma_\mu d_{1\mathrm{L}} - \bar{u}_{1\mathrm{L}}\gamma_\mu u_{1\mathrm{L}})\\ &\quad \left. -\sum_{\alpha=1}^{3}(\bar{e}^-_{\alpha\mathrm{L}}\gamma_\mu e^-_{\alpha\mathrm{L}} - \bar{\nu}_{e\alpha\mathrm{L}}\gamma_\mu \nu_{e\alpha\mathrm{L}})\right]\\ &= \sum_f T_{4f}\bar{f}_\mathrm{L}\gamma_\mu f_\mathrm{L},\\ J_{\mu,\mathrm{L}}(Z') &= (2T_\mathrm{W})^{-1}\left\{\sum_{j=2}^{3}\left[T_\mathrm{W}^2(\bar{u}_{a\mathrm{L}}\gamma_\mu u_{a\mathrm{L}} - \bar{d}_{a\mathrm{L}}\gamma_\mu d_{a\mathrm{L}})\right.\right.\\ &\quad \left. -\bar{D}_{a\mathrm{L}}\gamma_\mu D_{a\mathrm{L}} + \bar{U}_{a\mathrm{L}}\gamma_\mu U_{a\mathrm{L}}\right]\\ &\quad -T_\mathrm{W}^2(\bar{d}_{1\mathrm{L}}\gamma_\mu d_{1\mathrm{L}} - \bar{u}_{1\mathrm{L}}\gamma_\mu u_{1\mathrm{L}})\\ &\quad +\bar{U}_{1\mathrm{L}}\gamma_\mu U_{1\mathrm{L}} - \bar{D}_{1\mathrm{L}}\gamma_\mu D_{1\mathrm{L}}\\ &\quad +\sum_{\alpha=1}^{3}\left[-T_\mathrm{W}^2(\bar{e}^-_{\alpha\mathrm{L}}\gamma_\mu e^-_{\alpha\mathrm{L}} - \bar{\nu}_{\alpha\mathrm{L}}\gamma_\mu \nu_{\alpha\mathrm{L}})\right.\\ &\quad \left.\left. +\bar{N}^0_{\alpha\mathrm{L}}\gamma_\mu N^0_{\alpha\mathrm{L}} - \bar{E}^-_{\alpha\mathrm{L}}\gamma_\mu E^-_{\alpha\mathrm{L}}\right]\right\}\\ &= \sum_f T'_{4f}\bar{f}_\mathrm{L}\gamma_\mu f_\mathrm{L},\end{aligned} \tag{6}$$

where $T_{4f} = \mathrm{Dg}(1/2, -1/2, 0, 0)$ is the third component of the weak isospin and $T'_{4f} = (1/2T_\mathrm{W})\mathrm{Dg}(T_\mathrm{W}^2, -T_\mathrm{W}^2, -1, 1) = T_\mathrm{W}\lambda_3/2 + (1/T_\mathrm{W})(\lambda_8/(2\sqrt{3}) - \lambda_{15}/\sqrt{6})$ is a convenient 4×4 diagonal matrix, acting both of them on the representation 4 of $SU(4)_\mathrm{L}$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with (3) and (4).

The couplings of the mass eigenstates Z_1^μ and Z_2^μ are given by

$$\begin{aligned}H^\mathrm{NC} &= \frac{g}{2C_\mathrm{W}}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu\left[a_{i\mathrm{L}}(f)(1-\gamma_5)\right.\\ &\quad \left.+a_{i\mathrm{R}}(f)(1+\gamma_5)\right] f\}\\ &= \frac{g}{2C_\mathrm{W}}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu\left[g(f)_{iV} - g(f)_{iA}\gamma_5\right] f\},\end{aligned}$$

where

$$\begin{aligned}a_{1\mathrm{L}}(f) &= \cos\theta(T_{4f} - q_f S_\mathrm{W}^2)\\ &\quad +\frac{g'\sin\theta C_\mathrm{W}}{g}(T'_{4f} - q_f T_\mathrm{W}),\\ a_{1\mathrm{R}}(f) &= -q_f S_\mathrm{W}\left(\cos\theta S_\mathrm{W} + \frac{g'\sin\theta}{g}\right),\\ a_{2\mathrm{L}}(f) &= -\sin\theta(T_{4f} - q_f S_\mathrm{W}^2)\\ &\quad +\frac{g'\cos\theta C_\mathrm{W}}{g}(T'_{4f} - q_f T_\mathrm{W}),\\ a_{2\mathrm{R}}(f) &= q_f S_\mathrm{W}\left(\sin\theta S_\mathrm{W} - \frac{g'\cos\theta}{g}\right),\end{aligned} \tag{7}$$

and

$$\begin{aligned}g(f)_{1V} &= \cos\theta(T_{4f} - 2S_\mathrm{W}^2 q_f)\\ &\quad +\frac{g'\sin\theta}{g}(T'_{4f}C_\mathrm{W} - 2q_f S_\mathrm{W}),\\ g(f)_{2V} &= -\sin\theta(T_{4f} - 2S_\mathrm{W}^2 q_f)\\ &\quad +\frac{g'\cos\theta}{g}(T'_{4f}C_\mathrm{W} - 2q_f S_\mathrm{W}),\\ g(f)_{1A} &= \cos\theta T_{4f} + \frac{g'\sin\theta}{g}T'_{4f}C_\mathrm{W},\\ g(f)_{2A} &= -\sin\theta T_{4f} + \frac{g'\cos\theta}{g}T'_{4f}C_\mathrm{W}.\end{aligned} \tag{8}$$

The values of g_{iV}, g_{iA} with $i = 1, 2$ are listed in Tables 2 and 3.

Table 2. The $Z_1^\mu \longrightarrow \bar{f}f$ couplings

f	$g(f)_{1V}$	$g(f)_{1A}$
$u_{1,2,3}$	$\cos\theta\left(\frac{1}{2} - \frac{4S_\mathrm{W}^2}{3}\right) - \frac{5\sin\theta}{6(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$
$d_{1,2,3}$	$\left(-\frac{1}{2} + \frac{2S_\mathrm{W}^2}{3}\right)\cos\theta + \frac{\sin\theta}{6(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$	$-\frac{1}{2}\cos\theta - \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$
$D_{1,2,3}$	$\frac{2S_\mathrm{W}^2}{3}\cos\theta + \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}\left(\frac{7S_\mathrm{W}^2}{3} - 1\right)$	$-\frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}C_\mathrm{W}^2$
$U_{1,2,3}$	$-\frac{4S_\mathrm{W}^2}{3}\cos\theta - \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}\left(\frac{11S_\mathrm{W}^2}{3} - 1\right)$	$\frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}C_\mathrm{W}^2$
$e^-_{1,2,3}$	$\cos\theta\left(-\frac{1}{2} + 2S_\mathrm{W}^2\right) + \frac{5\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$	$-\frac{\cos\theta}{2} - \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$
$\nu_{1,2,3}$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}S_\mathrm{W}^2$
$N^0_{1,2,3}$	$\frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}C_\mathrm{W}^2$	$\frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}C_\mathrm{W}^2$
$E^-_{1,2,3}$	$2S_\mathrm{W}^2\cos\theta + \frac{\sin\theta}{(C_{2\mathrm{W}})^{1/2}}\left(2 - \frac{5}{2}C_\mathrm{W}^2\right)$	$-\frac{\sin\theta}{2(C_{2\mathrm{W}})^{1/2}}C_\mathrm{W}^2$

Table 3. The $Z_2^\mu \longrightarrow \bar{f}f$ couplings

f	$g(f)_{2V}$	$g(f)_{2A}$
$u_{1,2,3}$	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)-\frac{5\cos\theta}{6(C_{2W})^{1/2}}S_W^2$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$
$d_{1,2,3}$	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)\sin\theta+\frac{\cos\theta}{6(C_{2W})^{1/2}}S_W^2$	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$
$D_{1,2,3}$	$-\frac{2S_W^2}{3}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}\left(\frac{7S_W^2}{3}-1\right)$	$-\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$
$U_{1,2,3}$	$\frac{4S_W^2}{3}\sin\theta-\frac{\cos\theta}{2(C_{2W})^{1/2}}\left(\frac{11S_W^2}{3}-1\right)$	$\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$
$e^-_{1,2,3}$	$\sin\theta\left(\frac{1}{2}-2S_W^2\right)+\frac{5\cos\theta}{2(C_{2W})^{1/2}}S_W^2$	$\frac{\sin\theta}{2}-\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$
$\nu_{1,2,3}$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$
$N^0_{1,2,3}$	$\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$	$\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$
$E^-_{1,2,3}$	$-2S_W^2\sin\theta+\frac{\cos\theta}{(C_{2W})^{1/2}}\left(2-\frac{5}{2}C_W^2\right)$	$-\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$

As we can see, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

5 Fermion masses

The Higgs scalars introduced in Sect. 3 break the symmetry in an appropriate way. Now, in order to generate both a simple mass splitting between ordinary and exotic fermions and a consistent mass spectrum, we introduce an anomaly-free discrete Z_2 symmetry [6], with the following assignments of Z_2 charge q:

$$q(Q_{a\mathrm{L}}, u^c_{\alpha\mathrm{L}}, d^c_{\alpha\mathrm{L}}, L_{\alpha\mathrm{L}}, e^c_{\alpha\mathrm{L}}, \phi_1, \phi_3) = 0,$$
$$q(U^c_{\alpha\mathrm{L}}, D^c_{\alpha\mathrm{L}}, E^c_{\alpha\mathrm{L}}, \phi_2, \phi_4) = 1. \qquad (9)$$

Notice that ordinary fermions are not affected by this discrete symmetry.

The gauge invariance and the Z_2 symmetry allow for the following Yukawa lagrangians.

(1) For quarks:

$$\mathcal{L}_Y^Q = \sum_{j=2}^{3} Q^{\mathrm{T}}_{a\mathrm{L}} C \left\{ \phi_3^* \sum_{\alpha=1}^{3} h^u_{j\alpha} u^c_{\alpha\mathrm{L}} + \phi_4^* \sum_{\alpha=1}^{3} h^U_{j\alpha} U^c_{\alpha\mathrm{L}} + \phi_1 \sum_{\alpha=1}^{3} h^d_{j\alpha} d^c_{\alpha\mathrm{L}} + \phi_2 \sum_{\alpha=1}^{3} h^D_{j\alpha} D^c_{\alpha\mathrm{L}} \right\}$$
$$+ Q^{\mathrm{T}}_{1\mathrm{L}} C \left\{ \phi_1^* \sum_{\alpha=1}^{3} h^u_{1\alpha} u^c_{\alpha\mathrm{L}} + \phi_2^* \sum_{\alpha=1}^{3} h^U_{1\alpha} U^c_{\alpha\mathrm{L}} + \phi_3 \sum_{\alpha=1}^{3} h^d_{1\alpha} d^c_{\alpha\mathrm{L}} + \phi_4 \sum_{\alpha=1}^{3} h^D_{1\alpha} D^c_{\alpha\mathrm{L}} \right\} + \mathrm{h.c.},$$

where the $h's$ are Yukawa couplings and C is the charge conjugate operator.

(2) For charged leptons:

$$\mathcal{L}_Y^l = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} L^{\mathrm{T}}_{\alpha\mathrm{L}} C \left\{ \phi_3 h^e_{\alpha\beta} e^+_{\beta\mathrm{L}} + \phi_4 h^E_{\alpha\beta} E^+_{\beta\mathrm{L}} \right\} + \mathrm{h.c.}$$

The lagrangian $\mathcal{L}_Y^Q$ produces for up- and down-type quarks, in the basis $(u_1, u_2, u_3, U_1, U_2, U_3)$ and $(d_1, d_2, d_3, D_1, D_2, D_3)$ respectively, 6×6 block diagonal mass matrices of the form

$$M_{uU} = \begin{pmatrix} M_{u(3\times3)} & 0 \\ 0 & M_{U(3\times3)} \end{pmatrix},$$

where

$$M_u = \begin{pmatrix} h^u_{11}v & h^u_{21}v' & h^u_{31}v' \\ h^u_{12}v & h^u_{22}v' & h^u_{32}v' \\ h^u_{13}v & h^u_{23}v' & h^u_{33}v' \end{pmatrix},$$

$$M_U = \begin{pmatrix} h^U_{11}V & h^U_{21}V' & h^U_{31}V' \\ h^U_{12}V & h^U_{22}V' & h^U_{32}V' \\ h^U_{13}V & h^U_{23}V' & h^U_{33}V' \end{pmatrix},$$

and

$$M_{dD} = \begin{pmatrix} M_{d(3\times3)} & 0 \\ 0 & M_{D(3\times3)} \end{pmatrix},$$

where

$$M_d = \begin{pmatrix} h^d_{11}v' & h^d_{21}v & h^d_{31}v \\ h^d_{12}v' & h^d_{22}v & h^d_{32}v \\ h^d_{13}v' & h^d_{23}v & h^d_{33}v \end{pmatrix},$$

$$M_D = \begin{pmatrix} h^D_{11}V' & h^D_{21}V & h^D_{31}V \\ h^D_{12}V' & h^D_{22}V & h^D_{32}V \\ h^D_{13}V' & h^D_{23}V & h^D_{33}V \end{pmatrix}.$$

For the charged leptons the lagrangian $\mathcal{L}_Y^l$, in the basis $(e_1, e_2, e_3, E_1, E_2, E_3)$, also produces a block diagonal mass matrix

$$M_{eE} = \begin{pmatrix} M_{e(3\times3)} & 0 \\ 0 & M_{E(3\times3)} \end{pmatrix},$$

where the entries in the submatrices are given by

$$M_{e,\alpha\beta} = h^e_{\alpha\beta} v' \qquad \text{and} \qquad M_{E,\alpha\beta} = h^E_{\alpha\beta} V'.$$

The former mass matrices exhibit the mass splitting between ordinary and exotic charged fermions and show that all the charged fermions in the model acquire masses at the tree level. Clearly, by a judicious tuning of the Yukawa couplings and of the mass scales v and v', a consistent mass spectrum in the ordinary charged sector can be obtained. In the exotic charged sector all the particles acquire masses at the scale $V \sim V' \gg 174\,\text{GeV}$. Note that in the low energy limit our model corresponds to a Type III two Higgs doublet model [7] in which both doublets couple to the same type of fermions, with the quark and lepton couplings treated asymmetrically.

The neutral leptons remain massless as far as we use only the original fields introduced in Sect. 2. But as mentioned earlier, we may introduce one or more Weyl singlet states $N^0_{L,b}$, $b = 1, 2, \ldots$, which may implement the appropriate neutrino oscillations [8].

6 Constraints on the $(Z^\mu$–$Z'^\mu)$ mixing angle and the Z_2^μ mass

To bound $\sin\theta$ and M_{Z_2} we use parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table 4.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$ is

$$\Gamma(Z_1^\mu \to f\bar{f}) = \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\left\{\frac{3\beta-\beta^3}{2}\left[g(f)_{1V}\right]^2 + \beta^3\left[g(f)_{1A}\right]^2\right\} \times (1+\delta_f) R_{\text{EW}} R_{\text{QCD}}, \quad (10)$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C = 1$ for leptons while for quarks $N_C = 3(1 + \alpha_s/\pi + 1.405\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths, see [9]); R_{EW} is for the electroweak corrections which include the leading order QED corrections given by $R_{\text{QED}} = 1 + 3\alpha/(4\pi)$. R_{QCD} denotes further QCD corrections (for a comprehensive review, see [10] and references therein), and $\beta = \sqrt{1 - 4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark, for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}\left[-m_t^2/(2M_{Z_1}^2) + 1/5\right]$ [11]. The ρ parameter can be expanded as $\rho = 1 + \delta\rho_0 + \delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0 \approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree level contribution due to the $(Z_\mu$–$Z'_\mu)$ mixing which can be parametrized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2 - 1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which are listed in Table 2.

In what follows we are going to use the experimental values [12] $M_{Z_1} = 91.188\,\text{GeV}$, $m_t = 174.3\,\text{GeV}$, $\alpha_s(m_Z) = 0.1192$, $\alpha(m_Z)^{-1} = 127.938$, and $\sin^2\theta_W = 0.2333$. The experimental values are introduced using the definitions $R_\eta \equiv \Gamma(\eta\eta)/\Gamma(\text{hadrons})$ for $\eta = e, \mu, \tau, b, c$.

As a first result notice from Table 2, that our model predicts $R_e = R_\mu = R_\tau$, in agreement with the experimental results in Table 4.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W = -2\left[(2Z+N)c_{1u} + (Z+2N)c_{1d}\right], \quad (11)$$

where $c_{1q} = 2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the cesium atom is given by [13] $Q_W(^{133}_{55}\text{Cs}) = -73.09 \pm 0.04 + \Delta Q_W$, where the contribution of new physics is included in ΔQ_W, which can be written as [14]

$$\Delta Q_W = \left[\left(1 + 4\frac{S_W^4}{1-2S_W^2}\right)Z - N\right]\delta\rho_V + \Delta Q'_W. \quad (12)$$

The term $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{iA}$ and $g(q)_{iV}$, $i = 1, 2$, from Tables 2 and 3. The value we obtain is

$$\Delta Q'_W = (3.75Z + 2.56N)\sin\theta + (1.22Z + 0.41N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \quad (13)$$

The discrepancy between the SM and the experimental data for ΔQ_W is given by [15]

$$\Delta Q_W = Q_W^{\text{exp}} - Q_W^{\text{SM}} = 1.03 \pm 0.44, \quad (14)$$

which is 2.3 σ away from the SM predictions.

Introducing the expressions for Z pole observables in (10), with ΔQ_W in terms of new physics in (12) and using experimental data from LEP, SLC and atomic parity violation (see Table 4), we do a χ^2 fit and we find the best

Table 4. Experimental data and SM values for the parameters

	Experimental results	SM
Γ_Z (GeV)	2.4952 ± 0.0023	2.4966 ± 0.0016
$\Gamma(\text{had})$ (GeV)	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.019 ± 0.027
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21664 ± 0.00068	0.21569 ± 0.00016
R_c	0.1729 ± 0.0032	0.17230 ± 0.00007
Q_W^{Cs}	$-72.65 \pm 0.28 \pm 0.34$	-73.10 ± 0.03
M_{Z_1} (GeV)	91.1872 ± 0.0021	91.1870 ± 0.0021

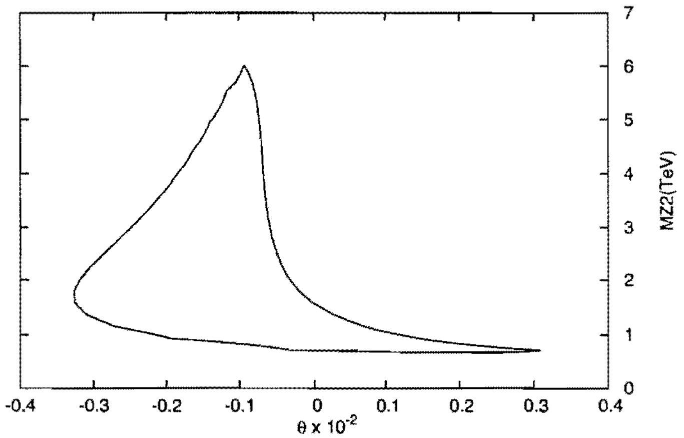

Fig. 1. Contour plot displaying the allowed region for θ versus M_{Z_2} at 95% C.L.

allowed region in the (θ–M_{Z_2}) plane at 95% confidence level (C.L.). In Fig. 1 we display this region, which gives us the constraints

$$-0.0032 \le \theta \le 0.0031, \quad 0.67\,\mathrm{TeV} \le M_{Z_2} \le 6.1\,\mathrm{TeV}. \tag{15}$$

As we can see, the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [16].

7 Conclusions

We have presented an anomaly-free model based on the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$, which does not contain exotic electric charges. This last constraint fixes the values $b = 1$ and $c = -2$ for the parameters in the electric charge generator in (1).

We break the gauge symmetry down to $SU(3)_c \otimes U(1)_Q$ in an appropriate way by using four different Higgs scalars ϕ_i, $i = 1, 2, 3, 4$, which set two different mass scales: $V \sim V' >> \sqrt{v^2 + v'^2} \simeq 174\,\mathrm{GeV}$, with $v \sim v'$. By introducing an anomaly-free discrete Z_2 symmetry we also obtain a simple mass splitting between exotic and ordinary fermions, and a consistent mass spectrum both in the quark and in the lepton sector. Notice also the consistence of our model in the charged lepton sector where it predicts the correct ratios R_η, $\eta = e, \mu, \tau$, in the Z decays. This is a characteristic feature of the two classes of three-family models introduced in [3].

By using experimental results we obtain a lowest bound of $0.67\,\mathrm{TeV} \le M_{Z_2}$ for the mass of an extra neutral gauge boson Z_2, and we find the bound of the mixing angle θ between the SM neutral current and the Z_2 one to be $-0.0032 < \theta < 0.0031$.

When we compare the numerical results presented in the previuos section with the results presented in [3], we find that the mixing angle θ is of the same order of magnitude ($\sim 10^{-3}$), but for the model considered here the mass associated with the new neutral current has smaller lower and upper bounds, with the lower bound just below the TeV scale, which allows for a possible signal at the Fermilab Tevatron.

For our analysis we have chosen just one of the two possible three-family models without exotic electric charges, characterized by the parameters $b = -c/2 = 1$ in the electric charge operator [3]. We believe that the low energy phenomenology for the other model must produce results similar to the ones presented in this paper.

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Anexo 3

SOFTWARE

Se incluyen en este anexo las rutinas usadas para hallar la región de confianza para el ángulo de mezcla y para la masa del nuevo bosón de gauge Z'^0 cuya existencia es predicha por los modelos estudiados en esta investigación. Este programa fue el usado para la obtención de las gráficas que aparecen en la penúltima sección de cada uno de los artículos publicados.

5.1. Programa principal

El siguiente es el programa principal que llama a la subrutina optimization.C

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <stdarg.h>
#include <stdlib.h>
#include "cfortran.h"
#include "minuit.h"

#include "optimization331.C"

main(void)
{
  FILE *ev,*ch;
  int i;
  double parameters[MAXPARAM],deltap[MAXPARAM];
  double bl[MAXPARAM],br[MAXPARAM],errors[MAXPARAM];
  double gamma;
  double chi;
  double teta,Mz2;
  double tini,tfin,dt,M2ini,M2fin,dM2;
  double tetap,tau,tauini,taufin,dtau;
```

```
  //**********************************************************
  //INITIALIZATION OF GLOBAL VARIABLES
  //**********************************************************
  OBLIQUE=(3*GF*MT*MT)/(8*M_PI*M_PI*sqrt(2));
  NC=3*(1+(0.1192/M_PI)+1.409*(0.1192/M_PI)*(0.1192/M_PI)-12.77
      *(0.1192/M_PI)*(0.1192/M_PI)*(0.1192/M_PI));

  //**********************************************************
  //READING EXPVAL FROM FILE
  //**********************************************************
  ev=fopen("experimental331.data","r");
  for(i=0;i<NUMBERQUANTITIES;i++){
    fscanf(ev,"%lf %lf",&expval[i],&sigma[i]);
  }
  fclose(ev);

  //**********************************************************
  //MINIMIZATION
  //**********************************************************
  //INTIAL GUESS FOR THETA
  parameters[0]=THETA_INI;
  deltap[0]=DELTA_THETA;
  bl[0]=BL_THETA;
  br[0]=BR_THETA;
  //INTIAL GUESS FOR MZ2
  parameters[1]=MZ2_INI;
  deltap[1]=DELTA_MZ2;
  bl[1]=BL_MZ2;
  br[1]=BR_MZ2;
  //MINIMIZATION
  FitChiSquare(ChiSquare,NUMPAR,parameters,deltap,bl,br,errors);

  //**********************************************************
  //CONSTRUCT CONFIDENCE REGION
  //**********************************************************

  //*
  ev=fopen("contourverbose.dat","r");
  fscanf(ev,"%lf %lf",&tini,&M2fin);
  fclose(ev);
  system("tail -n 1 contourverbose.dat > /tmp/cont.tmp");
  ev=fopen("/tmp/cont.tmp","r");
  fscanf(ev,"%lf %lf",&tfin,&M2ini);
  fclose(ev);

//MODIFIED
tini=-0.18;
M2fin=18.0;
tfin=0.16;
```

```
M2ini=0.7;

  //tini=-0.31;tfin=-0.25;
  dt=(tfin-tini)/30.0;
  //dtau=(taufin-tauini)/10.0;
  dM2=(M2fin-M2ini)/30.0;
  ch=fopen("chisquare.dat","w");
  for(teta=tini;teta<=tfin;teta+=dt){
    for(Mz2=M2ini;Mz2<=M2fin;Mz2+=dM2){
      parameters[0]=teta;
      parameters[1]=Mz2;
      ChiSquare(&chi,parameters);
      // fprintf(ch,"%10.10g \n",chi);
      fprintf(ch,"%lf %lf %lf\n",teta,Mz2,chi);
      //printf("%lf %lf %lf\n",teta,Mz2,chi);
    }
    fprintf(ch,"\n");
  }
  fclose(ch);
 }
```

A.2. Subrutina

En esta subrutina se usan librerias del CERN que nos permiten realizar la minimización de la χ^2

```
//====================================================

#define MAX(a,b) (a>b?a:b) //CONSTANT FUNCTION MAX, MIN
#define MIN(a,b) (a<b?a:b)

//Programation constants
#define MAXPARAM 30
#define NUMBERQUANTITIES 4
#define ORDERPARAM {3,1,0,2}
//                  | | | |
//                  B C L DQ
#define MAXQUANTITIES 15
#define MEDDATA 10000
#define NUMPAR 2

//Physical constants
//We take all this datas of the Particle Data Group (1 July 2002)
#define HVAR 6.58211889E-22
//The Planck constant divide by two times pi,  have units of: MeV.s.
#define GF  1.16639E-5
//The Fermi constant divide by hvar times c at three, have units of: GeV^{-2}
#define SW2 0.23113
#define CW2 0.76887
```

```
//Weinberg Square Sine and Cosine
#define THETA 0.292614162
#define MW 80.423
//W^{+ or -} mass give in GeV
#define MZ 91.1876
//Z_{1} mass give in GeV.
#define MB 4.25
//Bottom mass give GeV.
#define MT 174.30
//Top mass give GeV.
#define Z 55
//number of protons of the source Cesio (Cs)
#define NN 78
//number of neutrons of the source Cesio (Cs)

//VALUES FOR MINIMIZATION
//MIXING ANGLE
//*POSITIVE REGION
#define THETA_INI -3//x 10^-2
#define DELTA_THETA 1
#define BL_THETA -1e2  //LARGER THAN ZERO TO AVOID SINGULARITY
#define BR_THETA 1e2
#define SCALE_THETA 1e-2
//*/
/*POSTIVE REGION
  #define THETA_INI 3//x 10^-2 ^+
#define DELTA_THETA 1
#define BL_THETA -1e2  //LARGER THAN ZERO TO AVOID SINGULARITY
#define BR_THETA 1e2
#define SCALE_THETA 1e-2
//*/
//Z2 MASS
//*WHEN MASS Z2 IS USED
#define MZ2_INI 3
#define DELTA_MZ2 0.05
#define BL_MZ2 0.7
#define BR_MZ2 1e3
#define SCALE_MZ2 1e3 //GEV
//*/
//*WHEN TREE IS USED
#define TAU_INI 5
#define DELTA_TAU 1//0.05
#define BL_TAU 0.0
#define BR_TAU 1e2
#define SCALE_TAU 1e0 //GEV
//*/


//=====================================================
```

```
//Number of Colors
double OBLIQUE,NC;
double expval[MAXQUANTITIES],theo[MAXQUANTITIES],sigma[MAXQUANTITIES];

       //COMPUTES THE g FUNCTION FOR ONE FERMION
       //FERMION TABLE:
//=============
       //1: u_{1,2},2: d_{1,2},3: D_{1,2},4:D'_{1,2},5:
       //d_{3},6:u_{3},7:U_{3},8:U'_{3},9:e-_{1,2,3}
       //10:v_{1,2,3},11:N0_{1,2,3},12:N0'_{1,2,3}
void ge(double theta,int fermion,double *g1V,double *g1A)
{  // double THETA;
  //THETA=sqrt(SW2*CW2)/sqrt(3.0-4.0*SW2);
 switch(fermion){
  case 1:
    // u_{1,2}
    //*g1V=(1.0/2.0-(4.0*SW2/3.0))*(cos(theta)-(sin(theta)/sqrt(3.0-4.0*SW2)));
    //*g1A=(1.0/2.0)*(cos(theta)-(sin(theta)/sqrt(3.0-4.0*SW2)));
    //*********************************************************************
     //u_{2,3}
    *g1V=(1.0/2.0-(4.0*SW2/3.0))*cos(theta)-THETA*((1.0/(2.0*sqrt(CW2*SW2)))
           -(4.0*sqrt(SW2/CW2)/3.0))*sin(theta);
   *g1A=(1.0/2.0)*cos(theta)-THETA*(1.0/(2.0*sqrt(CW2*SW2)))*sin(theta);
    //I do only one change...in the above expression...
   break;
  case 2:
    //d_{1,2}
    // *g1V=(-1.0/2.0+(2.0*SW2/3.0))*cos(theta)+
    // (2.0*SW2-3.0)*(sin(theta)/(6.0*sqrt(3.0-4.0*SW2)));
   // *g1A=(-1.0/2.0)*(cos(theta)+(sin(theta)*(1-2.0*SW2)/sqrt(3.0-4.0*SW2)));
     //*******************************************************************
     //d_{2,3}
     *g1V=(-1.0/2.0+(2.0*SW2/3.0))*cos(theta)-THETA*sin(theta)
           *((CW2-SW2)/(2.0*sqrt(CW2*SW2)))-
            THETA*sin(theta)*(2.0/3.0)*sqrt(SW2/CW2);
     *g1A=(-1.0/2.0)*cos(theta)
          -sin(theta)*THETA*((CW2-SW2)/(2.0*sqrt(CW2*SW2)));
      //I do only one change...in the above expression...
     break;
  case 3:
    //D_{1,2}
    // *g1V=(2.0*SW2*cos(theta)/3.0)
    // +(sin(theta)*(3.0-5.0*SW2)/3.0*sqrt(3.0-4.0*SW2));
    // *g1A=CW2*sin(theta)/sqrt(3.0-4.0*SW2);
           //***********new data****************
  *g1V=((2.0*SW2*cos(theta))/3.0)
        +sin(theta)*THETA*(sqrt(CW2/SW2)+(2.0*sqrt(SW2/CW2))/3.0);
    *g1A=-THETA*sqrt(CW2/SW2)*sin(theta);
    break;
  case 4:
```

```
  //d_{3}
  // *g1V=(-1.0/2.0+(2.0*SW2)/3.0)*cos(theta)+
        //(sqrt(3.0-4.0*SW2)*sin(theta)/6.0);
  // *g1A=(-1.0/2.0)*(cos(theta)-(sin(theta)/sqrt(3.0-4.0*SW2)));
        //*****************************
  //Now this is d_{1}
   *g1V=(-1.0/2.0+((2.0*SW2)/3.0))*cos(theta)
     -THETA*sin(theta)*((1.0/(2.0*sqrt(CW2*SW2)))
          -(2.0*sqrt(SW2/CW2)/3.0));
   *g1A=(-1.0/2.0)*cos(theta)-THETA*(1.0/(2.0*sqrt(CW2*SW2)))*sin(theta);
   break;
 case 5:
   //u_{3}
  // *g1V=cos(theta)*(1.0/2.0-(4.0*SW2)/3.0)+
       // ((2.0*SW2+3.0)*sin(theta)/6.0*sqrt(3.0-4.0*SW2));
 //*g1A=(1.0/2.0)*cos(theta)+(sin(theta)*(1-2.0*SW2)/sqrt(3.0-4.0*SW2));
   //Now this is u_{1}
 *g1V=cos(theta)*(1.0/2.0-((4.0*SW2)/3.0))
      -sin(theta)*THETA*(((CW2-SW2)/(2.0*sqrt(CW2*SW2)))
         +((2.0*sqrt(SW2/CW2))/3.0));
 *g1A=(1.0/2.0)*cos(theta)-sin(theta)*THETA*((CW2-SW2)/(2.0*sqrt(CW2*SW2)));
  break;
 case 6:
   //U
   // *g1V=(4.0/3.0)*(-SW2*cos(theta)
   //+(((3.0/2.0)-4.0*SW2)*sin(theta)/sqrt(3.0-4.0*SW2)));
   // *g1A=-(1.0/3.0)*sqrt(3.0-4.0*SW2)*sin(theta);
   //********the new form of the expressions******
   *g1V=(-4.0/3.0)*SW2*cos(theta)
       -THETA*sin(theta)*(sqrt(CW2/SW2)+(4.0*sqrt(SW2/CW2))/3.0);
   *g1A=THETA*sin(theta)*sqrt(CW2/SW2);
   break;
case 7:
   //e-_{1,2,3}
   // *g1V=cos(theta)*(-(1.0/2.0)+2.0*SW2)
   // -((sin(theta)*(1.0-4.0*SW2))/2.0*sqrt(3.0-4.0*SW2));
   // *g1A=-(1.0/2.0)*(cos(theta)-(sin(theta)/sqrt(3.0-4.0*SW2)));
   //*******the new leptons********
   *g1V=cos(theta)*(-(1.0/2.0)+2.0*SW2)
     +THETA*sin(theta)*((1.0/(2.0*sqrt(CW2*SW2)))
                       -2.0*sqrt(SW2/CW2));
   *g1A=-(1.0/2.0)*cos(theta)+THETA*(1.0/(2.0*sqrt(CW2*SW2)))*sin(theta);
   //I do only one change...in the above expression...
  break;
 case 8:
//v_{1,2,3}
   // *g1V=(1.0/2.0)*(cos(theta)
           //+(sin(theta)*(1.0-2.0*SW2)/sqrt(3.0-4.0*SW2)));
   // *g1A=(1.0/2.0)*(cos(theta)
          //+(sin(theta)*(1.0-2.0*SW2)/sqrt(3.0-4.0*SW2)));
```

```
    //**************the new neutrinos********
     //v_{1,2,3}
     *g1V=(1.0/2.0)*cos(theta)-sin(theta)*THETA*((CW2-SW2)/(2.0*sqrt(CW2*SW2)));
     *g1A=(1.0/2.0)*cos(theta)-sin(theta)*THETA*((CW2-SW2)/(2.0*sqrt(CW2*SW2)));
     break;
   case 9:
      //NO_{1,2,3}
     // *g1V=(-1.0/6.0)*sqrt(3.0-4.0*SW2)*sin(theta);
     //*g1A=(-1.0/6.0)*sqrt(3.0-4.0*SW2)*sin(theta);
     //**************+the new exotic scalars
     //NO_{1,2,3}
     *g1V=-THETA*sqrt(CW2/SW2)*sin(theta);
     *g1A=-THETA*sqrt(CW2/SW2)*sin(theta);
     break;
   }
}

//COMPUTATION OF THE Z DECAY WIDTH
double Gamma(double parameters[],int fermion)
{
  double gamZ;
  double teta,Mz2;
  double g1V,g1A;
  double TREE,BETHA,DELTB,QCD,QED;

  //*******************************************************
  //READ OF THE PARAMETERS ARRAY
  //*******************************************************
  teta=parameters[0];//teta REPRESENT THETA
  Mz2=parameters[1];//Mz2 IS THE MASS OF THE NUETRAL BOSON 2
  TREE=(((Mz2*Mz2)/(MZ*MZ))-1.0)*sin(teta)*sin(teta);
  BETHA=sqrt(1.0-((4*MB*MB)/(MZ*MZ)));
  DELTB=(1.0/100)*(-((MT*MT)/(2*MZ*MZ))+(1.0/5));
  QCD=1.0;
  QED=1.0+(3/(4*M_PI*127.938));

  switch(fermion){
  case 0://GAMMA BOTTOM-ANTIBOTOM
    ge(teta,2,&g1V,&g1A);
    gamZ=((NC*GF*MZ*MZ*MZ)/(6*M_PI*sqrt(2)))*(1+OBLIQUE+TREE)*
(((3*BETHA-BETHA*BETHA*BETHA)/2)*g1V*g1V+BETHA*BETHA*BETHA*g1A*g1A)*
(1+DELTB)*(QCD*QED);
    break;
  case 1://GAMMA CHARM-ANTICHARM
    ge(teta,1,&g1V,&g1A);
    gamZ=((NC*GF*MZ*MZ*MZ)/(6*M_PI*sqrt(2)))*(1+OBLIQUE+TREE)*
((g1V*g1V)+(g1A*g1A))*1*(QCD*QED);
    break;
  case 2://GAMMA LEPTON-ANTILEPTON
    ge(teta,7,&g1V,&g1A);
```

```
    gamZ=((1*GF*MZ*MZ*MZ)/(6*M_PI*sqrt(2)))*(1+OBLIQUE+TREE)
*((g1V*g1V)+(g1A*g1A))*1*(QCD*QED);
    break;
  }
  return gamZ;
}

double DeltaNucleusCharge(double parameters[])
{ double teta,Mz2;
  double tree,DQ;
  double ae,vu,vd,Ae,Vu,Vd;
        //These are the g values (Axial y Vectorial) of the  Z1 y Z2,
        //repectively in the Theta Zero Limit.
  teta=parameters[0];
  Mz2=parameters[1];
  tree=(((Mz2*Mz2)/(MZ*MZ))-1)*sin(teta)*sin(teta);
        //Our 3-3-1 Model with minmal scalar sector
        //Theta Zero Limit of constants of the Z1
        //ae=+(1.0/2.0);//3-3-1 Model of Long
  ae=-(1.0/2.0);
  vu=(1.0/2.0)-(4.0*SW2/3.0);
  vd=-(1.0/2.0)+(SW2/3.0);
        //Theta Zero Limit of constants of the Z2
        //Ae=(1.0/2.0)*(1.0/sqrt(3.0-4.0*SW2));
        //Vu=-((1.0/2.0)-(4.0*SW2/3.0))*(1.0/sqrt(3.0-4.0*SW2));
        //Vd=-((1.0/2.0)-(SW2/3.0))*(1.0/sqrt(3.0-4.0*SW2));
        //this is the new constants which were obtained by William Professor
        //Theta zero limit which imply Phi zero limit and Psi zero limit
   Ae=-THETA/(2.0*sqrt(SW2*CW2));
   Vu=-THETA*(((CW2-SW2)/(2.0*sqrt(SW2*CW2)))+(4.0*sqrt(SW2/CW2)/3.0));
   Vd=-THETA*((1.0/(2.0*sqrt(CW2*SW2)))-(2.0*sqrt(SW2/CW2))/3.0);
        //Delta Q
   DQ=((1+((4*SW2*SW2)/(1.0-2*SW2)))*Z-NN)*tree;
   DQ+=16*((2.0*Z+NN)*(ae*Vu+Ae*vu)+(Z+2.0*NN)*(ae*Vd+Ae*vd))*sin(teta);
   DQ+=16*((2.0*Z+NN)*Ae*Vu+(Z+2.0*NN)*Ae*Vd)*((MZ*MZ)/(Mz2*Mz2));
  return DQ;
}


//****************************************************************
//THEORETICAL VALUES OF QUANTITIES
double TheoreticalValues(int i,double parameters[])
{

  int j;
  double theo;
  double order[]=ORDERPARAM;

  if(i==order[0]){
       //GAMMA BOTTOM-ANTIBOTTOM
```

```
    theo=Gamma(parameters,0);
  }
  else if(i==order[1]){
        //GAMMA CHARM-ANTICHARM
    theo=Gamma(parameters,1);
  }
  else if(i==order[2]){
        //GAMMA LEPTONS(WE USED THE UNIVERSALITY-ELECTRON-ANTIELECTRON)
    theo=Gamma(parameters,2);
  }
  else if(i==order[3]){//CESIO Q
        // theo=NucleusCharge(parameters);
        //is better DQ since is a quantitie of the minor order
    theo=DeltaNucleusCharge(parameters);
  }

  return theo;

}

//CHISQUARE COMPUTATION
void ChiSquare(double *chisq,double parameters[])
{
  int i;
  double inparam[MAXPARAM];

  //CONVERSION OF PARAMETERS TO ITS REAL SCALE
  inparam[0]=asin(parameters[0]*SCALE_THETA);
  inparam[1]=parameters[1]*SCALE_MZ2;
  //inparam[1]=parameters[1]*SCALE_TAU;

  *chisq=0;
  for(i=0;i<NUMBERQUANTITIES;i++){
    theo[i]=TheoreticalValues(i,inparam);
    *chisq+=(expval[i]-theo[i])*(expval[i]-theo[i])/(sigma[i]*sigma[i]);
  }

  //TAKE INTO ACCOUNT THAT INSIDE CHISQUARE MASS IS COMPUTED IN GEV
  //AND THE VALUE REPORTED MUST BE IN TEV
  parameters[1]=inparam[1]/SCALE_MZ2;
  //parameters[1]=inparam[1]/SCALE_TAU;
}

void shcmd(int n,...)
{
  FILE *scr;
  char *opcion;
  int i;

  va_list vara;
```

```
  va_start(vara,n);

  scr=fopen("/tmp/shcmd-diego","a");

  for(i=1;i<=n;i++){
    opcion=va_arg(vara,char *);
    fprintf(scr,"%s",opcion);
  }

  fprintf(scr,"\n");

  fclose(scr);

  system("source /tmp/shcmd-diego");
  system("rm -rf /tmp/shcmd-diego");
}

int countfile(char *file)
{
  FILE *cn;
  int num;

  shcmd(3,"wc -l ",file," | awk '{print $1}' > /tmp/count-diego.tmp");

  cn=fopen("/tmp/count-diego.tmp","r");
  fscanf(cn,"%d",&num);
  fclose(cn);

  shcmd(1,"rm -rf /tmp/count-diego.tmp");

  return num;
}

//SPECIAL FUNCTION FOR MIGRAD OPTIMIZATION
void OptimizationFunction(int npar, double grad[],
                          double *fcnval,double xval[],
                          int iflag, void (*Dummy)())
{
  int i;
  FILE *ts;

  switch(iflag) {
  case 1:
    break;
  case 2:
    break;
  default:
    (*Dummy)(fcnval,xval);
    //STORE VALUES IN VERBOSE FILE
```

```
    ts=fopen("chiverbose.dat","a");
    for(i=0;i<NUMPAR;i++) fprintf(ts,"%lf ",xval[i]);
    for(i=0;i<NUMBERQUANTITIES;i++) fprintf(ts,"%lf ",theo[i]);
    fprintf(ts,"%lf\n",*fcnval);
    fclose(ts);

    break;
  }
}

//ROUTINE TO FIT PARAMETERS BY CHISQUARE
void FitChiSquare(void (*funcion)(),int npar,double param[],double deltap[]
                 ,double bl[],double br[],double errors[])
{
  int i;
  double cmd_param[MAXPARAM];
  double tmp;
  int error_flag;
  char name[10];
  char *nums[]={"1","2","3","4","5"};
  char *command;

  //ERASE VERBOSE FILES
  system("rm -rf chiverbose.dat");

  //INITIALIZE ROUTINE MINUIT
  MNINIT(5,6,7);

  //SET TITLE OF MINIMIZATION
  command="Chi-Square Analysis";
  MNSETI(command);

  //SET PARAMETERS
  for(i=0;i<npar;i++){
    command="par";
    MNPARM(i+1,command,param[i],deltap[i],bl[i],br[i],error_flag);
  }

  //SET SOME CONSTANTS
  cmd_param[0]=0.5;
  command="SET ERRORDEF";
  MNEXCM(OptimizationFunction,command,cmd_param,1,error_flag,funcion);
  cmd_param[0]=2;
  command="SET STRATEGY";
  MNEXCM(OptimizationFunction,command,cmd_param,1,error_flag,funcion);
  cmd_param[0]=2;
  command="SET PRINTOUT";
  MNEXCM(OptimizationFunction,command,cmd_param,1,error_flag,funcion);

  //CALL SIMPLEX
```

```
command="SIMPLEX";
MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);
//CALL MIGRAD
command="MIGRAD";
MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);

//STORE MINIMIZATION SEARCH
system("mv chiverbose.dat minverbose.dat");

//SCAN PARAMETERS INTERMEDIATE
for(i=0;i<npar;i++){
  if(deltap[i]){
    cmd_param[0]=i+1;
    cmd_param[1]=40;
    cmd_param[2]=bl[i];
    cmd_param[3]=br[i];
    command="SCAN";
    MNEXCM(OptimizationFunction,command,cmd_param,4,error_flag,funcion);
  }
}
command="HESSE";
MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);
//RECALL MIGRAD
command="MIGRAD";
MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);

//RETURN PARAMETERS RESULT
for(i=0;i<npar;i++){
  MNPOUT(i+1,name,param[i],tmp,tmp,tmp,error_flag);
  MNERRS(i+1,tmp,tmp,errors[i],tmp);
}

//STORE MINIMIZATION SEARCH
system("mv chiverbose.dat minverbose.dat");

//SCAN PARAMETERS
for(i=0;i<npar;i++){
  if(deltap[i]){
    cmd_param[0]=i+1;
    cmd_param[1]=40;
    cmd_param[2]=param[i]-deltap[i];
    cmd_param[3]=param[i]+deltap[i];
    command="SCAN";
    MNEXCM(OptimizationFunction,command,cmd_param,4,error_flag,funcion);
    //STORE SCAN VERBOSE
    shcmd(3,"mv chiverbose.dat scanverbose-",nums[i],".dat");
  }
}

//CONTOUR
```

```
  //*
  cmd_param[0]=0.5;
  command="SET ERRORDEF";
  MNEXCM(OptimizationFunction,command,cmd_param,1,error_flag,funcion);
  cmd_param[0]=1;cmd_param[1]=2;
  command="CONTOUR";
  MNEXCM(OptimizationFunction,command,cmd_param,2,error_flag,funcion);
  //STORE CONTOUR VERBOSE
  system("mv chiverbose.dat contourverbose.dat");
  //*/

  command="SHOW FCN";
  MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);
  command="SHOW COV";
  MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);
  command="SHOW EIG";
  MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);
  command="SHOW COR";
  MNEXCM(OptimizationFunction,command,0,0,error_flag,funcion);

  //PRINT RESULTS
  printf("\n\n");
  for(i=0;i<npar;i++){
    printf("parameter %d = %lf +/- %lf\n",i,param[i],errors[i]);
  }
  printf("\n\n");

}

//PRINTS A DATA FILE USING COLOR LEVEL CODES
//colx,coly,colz DATA COORDINATES (IF COLZ=0 PLOT (COLX,COLY))
void levelplot(char *file,int ndata,int cols,int colx,int coly,
int colz,int colh int qlevel,char *plotfile,int rotz,int rotx,int graphflag)
{
  int i,j;
  int dim;
  double hmax,hmin,deltah;
  double val;
  FILE *fl,*gp;
  char *pl[]={"plot","splot"};
  double xx[MEDDATA],yy[MEDDATA],zz[MEDDATA],hh[MEDDATA];
  int levels[7]={3,5,2,7,4,1,6};

  int color,cindex;

  dim=colz?1:0;//1: 3D, 0:2D

  fl=fopen(file,"r");
  for(i=0;i<ndata;i++){
    for(j=1;j<=cols;j++){
```

```
      fscanf(fl,"%lf",&val);
      if(j==colx) xx[i]=val;
      if(j==coly) yy[i]=val;
      if(j==colz) zz[i]=val;
      if(j==colh) hh[i]=val;
    }
    hmax=i==0?hh[i]:MAX(hh[i],hmax);
    hmin=i==0?hh[i]:MIN(hh[i],hmin);
  }
  fclose(fl);

  gp=fopen(plotfile,"w");
  fprintf(gp,"set view %d,%d\n%s",rotz,rotx,pl[dim]);
  deltah=6.0/((hmax-hmin)/qlevel);
  for(i=0;i<ndata;i++){
    cindex=(int)(deltah*(hh[i]-hmin));
    color=levels[cindex%7];
    fprintf(gp," '-' not w lp %d",color);
    if(i<ndata-1) fprintf(gp,",");
  }
  fprintf(gp,"\n");
  for(i=0;i<ndata;i++){
    fprintf(gp,"%lf %lf ",xx[i],yy[i]);
    if(dim) fprintf(gp,"%lf",zz[i]);
    fprintf(gp,"\ne\n");
  }
  fprintf(gp,"pause -1 \"Press enter to continue...\"");
  fclose(gp);
  if(graphflag) system("gnuplot level-plot.gpl");
}
```

6. Informe financiero

Se adjuntan aquí el informe de ejecución presupuestal acumulada, con fecha del 25 de Abril de 2005, expedido por la DIME, la carta de la Directora de la DIME del 23 de Febrero de 2005 en la que me informa de un saldo por ejecutar en el proyecto de $ 958.343.00, y mi respuesta en la que solicito invertir ese saldo en la importación de dos libros.

SFI - Sistema Financiero Integrado
Módulo Presupuesto

UNIVERSIDAD NACIONAL DE COLOMBIA
SEDE MEDELLIN
Ejecución Presupuestal Acumulada por Proyecto y Recurso
3001 NIVEL CENTRAL SEDE MEDELLIN
899999063

Desde 200401 hasta 200411

25/04/2005 15:12:45
Programa: PYMREAPP
USUARIO 3 DEL CINDEC-D

Proyecto: 20101004541-MODELOS DE UNIFICACIÓN DE INTERACCIONES FUNDAMENTALES BASADOS EN EL GRUPO DE SIMETRÍA SU(3)C X SU(4)L X U(1)X

Recurso: 98 RECURSOS DE CAPITAL

Imputación	Descripción	Apropiación Definitiva	Cupo	Disponibilidad	Registro	Obligaciones	Pago	Saldo x Comprometer	%por Eje
2	***TOTAL: GASTOS***	***8,293,824.00***	***.00***	***7,371,469.00***	***7,371,469.00***	***3,780,720.00***	***2,522,909.00***	***922,355.00***	***11.12***
24	***INVERSION***	***8,293,824.00***	***.00***	***7,371,469.00***	***7,371,469.00***	***3,780,720.00***	***2,522,909.00***	***922,355.00***	***11.12***
24410	***Investigacion aplicada a estudios***	***8,293,824.00***	***.00***	***7,371,469.00***	***7,371,469.00***	***3,780,720.00***	***2,522,909.00***	***922,355.00***	***11.12***
24410705	***Educacion superior***	***8,293,824.00***	***.00***	***7,371,469.00***	***7,371,469.00***	***3,780,720.00***	***2,522,909.00***	***922,355.00***	***11.12***
2441070501	***PROGRAMA DE DESARROLLO INVESTIGATIVO***	***8,293,824.00***	***.00***	***7,371,469.00***	***7,371,469.00***	***3,780,720.00***	***2,522,909.00***	***922,355.00***	***11.12***
244107050101	***SERVICIOS PERSONALES INDIRECTOS***	***2,513,160.00***	***.00***	***1,887,018.00***	***1,887,018.00***	***1,258,012.00***	***1,258,012.00***	***626,142.00***	***24.91***
244107050101002	***SERVICIOS PERSONALES INDIRECTOS ASOCIADOS A CONTRATOS, PROGRAMAS, Y/O PROYECTOS***	***2,513,160.00***	***.00***	***1,887,018.00***	***1,887,018.00***	***1,258,012.00***	***1,258,012.00***	***626,142.00***	***24.91***
24410705010100203	Remuneracion por Servicios Tecnicos	2,513,160.00	.00	1,887,018.00	1,887,018.00	1,258,012.00	1,258,012.00	626,142.00	24.91
244107050102	***GASTOS DE OPERACION***	***5,780,664.00***	***.00***	***5,484,451.00***	***5,484,451.00***	***2,522,708.00***	***1,264,897.00***	***296,213.00***	***5.12***
244107050102001	***ADQUISICION DE BIENES***	***4,920,281.00***	***.00***	***4,717,786.00***	***4,717,786.00***	***1,870,442.00***	***612,631.00***	***202,495.00***	***4.12***
24410705010200101	COMPRA DE EQUIPO	4,140,000.00	.00	4,105,155.00	4,105,155.00	1,257,811.00	0.00	34,845.00	.84
24410705010200102	MATERIALES Y SUMINISTROS	780,281.00	.00	612,631.00	612,631.00	612,631.00	612,631.00	167,650.00	21.49
244107050102002	***ADQUISICION DE SERVICIOS***	***860,383.00***	***.00***	***766,665.00***	***766,665.00***	***652,266.00***	***652,266.00***	***93,718.00***	***10.89***
24410705010200205	IMPRESOS Y PUBLICACIONES	860,383.00	.00	766,665.00	766,665.00	652,266.00	652,266.00	93,718.00	10.89

CLAUDIA PATRICIA CASTAÑO ALZATE
PRESUPUESTO

EDGAR CADAVID CALDERON
PRESUPUESTO

UNIVERSIDAD NACIONAL DE COLOMBIA

SFI - Sistema Financiero Integrado
Módulo Presupuesto

UNIVERSIDAD NACIONAL DE COLOMBIA
SEDE MEDELLIN
Ejecución Presupuestal Acumulada por Proyecto y Recurso
3001 NIVEL CENTRAL SEDE MEDELLIN
899999063

Desde 200501 hasta 200504

25/04/2005 15:13:26
Programa: PYMREAPP
USUARIO 3 DEL CINDEC-D

Proyecto: 20101004541-MODELOS DE UNIFICACIÓN DE INTERACCIONES FUNDAMENTALES BASADOS EN EL GRUPO DE SIMETRÍA SU(3)C X SU(4)L X U(1)X

Recurso: 21 RECURSOS DE CAPITAL

Imputación	Descripción	Apropiación Definitiva	Cupo	Disponibilidad	Registro	Obligaciones	Pago	Saldo x Comprometer	%por Eje
2	*TOTAL: GASTOS*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
24	*INVERSION*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
24410	*Investigacion aplicada a estudios*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
24410705	*Educacion superior*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
2441070501	*PROGRAMA DE DESARROLLO INVESTIGATIVO*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
244107050102	*GASTOS DE OPERACION*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
244107050102002	*ADQUISICION DE SERVICIOS*	*958,343.00*	*.00*	*526,821.00*	*526,821.00*	*438,434.00*	*438,434.00*	*431,522.00*	*45.03*
24410705010200205	IMPRESOS Y PUBLICACIONES	958,343.00	.00	526,821.00	526,821 00	438,434.00	438,434.00	431,522.00	45.03

CLAUDIA PATRICIA CASTAÑO ALZATE
PRESUPUESTO

EDGAR CADAVID CALDERON
PRESUPUESTO

DIME

DIME-20

Medellín, Febrero 23 de 2005

Profesor
LUIS ALBERTO SÁNCHEZ DUQUE
Facultad de Ciencias
Escuela de Física
La Sede

Referencia: Proyecto de Investigación: *"Modelos de unificación de interacciones fundamentales basados en el grupo de simetría SU(3)C X SU(4)L X U(1)X"*

Apreciado Investigador:

Se esta remitiendo el saldo disponible para el proyecto en mención.

Por Consejo de Sede se aprobó pasar solamente los saldos reportados al 30 de noviembre, superiores a un salario mínimo y gravar con el 5% la totalidad de dichos saldos para proveer de apoyo administrativo financiero para todos los trámites.

Solicitamos diligenciar los cambios que sean estrictamente necesarios y reportarlos por medio de las Coordinaciones de cada facultad, a esta dependencia, hasta el **viernes próximo (25 de febrero) a las 12:00 del día**; debido a que el sistema financiero sólo recibe modificaciones cada dos meses y esa es la fecha establecida por cronograma y cierre automático del sistema.

Si no se van a utilizar los recursos pendientes debe notificarse inmediatamente a la Coordinación de cada facultad.

Cod. Quipú	Rubro	Saldo
20101004541	Remun. Servicios Técnicos	610.548
	Materiales y Suministros	167.650
	Impresos y Publicaciones	180.145
Valor total		**958.343**

Cordialmente,

OLGA CECILIA GUZMÁN MORALES
Directora DIME

Medellín, Febrero 24 de 2005

Profesora
OLGA CECILIA GUZMÁN MORALES
Directora DIME
Universidad Nacional de Colombia – sede Medellín
La Sede

Apreciada Profesora,

con referencia a los recursos pendientes del proyecto de investigación *"Modelos de unificación de interacciones fundamentales basados en el grupo de simetría SU(3)c X SU(3)L X U(1)X"*, del cual soy investigador principal, me permito solicitarle cambio de rubro en el sentido que todos esos recursos ($ 958.343) sean invertidos en la importación de los libros que listo a continuacuón:

1. Título: Strings, Branes and Gravity.
   Autor: Theoretical Advanced Study Institute in Elementary Particle Physics, et al.
   Página web: www.amazon.com
   Precio (en dólares): 134.00

2. Título: Unification and Supersymmetry: the frontiers of quark-lepton physics.
   Autor: R. N. Mohapatra.
   Editorial: Springer-Verlag.
   Página web: www.amazon.com
   Precio (en dólares): 59.50

Cordialmente,

____

LUIS ALBERTO SÁNCHEZ DUQUE
Profesor, Escuela de Física

Vo.Bo. ______________________________

DIEGO MEJÍA DUQUE
Vicedecano (E) de Investigaciones
Facultad de Ciencias

UNIVERSIDAD NACIONAL DE COLOMBIA SEDE MEDELLÍN
Programas Curriculares en Física
Facultad de Ciencias

PCF-078

Medellín, junio 10 de 2005

A QUIEN CORRESPONDA:

La Coordinadora de Programas Curriculares de la Escuela de Física y de la Maestría en Física, se permite dar constancia que el docente LUIS ALBERTO SÁNCEHZ DUQUE adscrito a la Escuela de Física, participó como expositor en el seminario permanente de la Maestría en Física.

Conferencia: *"La Extensión 3-3-1 del modelo estándar de las interacción fundamentales"*

Fecha: Viernes 3 de septiembre de 2004

Hora: 10:00 a.m.

Lugar: Auditorio Samuel Melguizo

Cordialmente,

Claudia García G.

CLAUDIA GARCÍA GARCÍA
Coordinadora
Programas Curriculares Escuela de Física

Laura T.

"30 años de cultura científica"

Autopista Norte Calle 64 x Carrera 65, Apartado 3308 –Teléfono: 430 98 91 Fax: 260 44 89 Medellín